Overview of New Equity Structured Products

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OUTLINE

- A Survey of the Products.

- Rationale for the Products.

- A Unified view of the Products and their Rationale.

Common Components for the Products

- The products involve a notional amount in a specific currency that scales the payoffs, quantoed if necessary.
- There is an effective date for the start of the product.
- There is a final date or maturity for the product.
- There are a set of underliers whose values on specified observation dates determines the payoff.
- The observation dates have a regular frequency that could be daily, monthly, quarterly or annual.
- Early termination is possible when a prespecified condition is met.

- Separate payoff rules on early termination.

- Payoff schedules in the absence of termination.

- The product may be quoted with an up front price, an up front option premium, or a zero cost of entry, swap structure.

Classification of Products

- Trigger Redeemable Notes.
- Vanilla Arithmetic and Product Cliquets.
- Swing Cliquets.
- Napoleonic Features.
- Incorporating Lock-Ins.
- Intermediate floors and caps.
- Melting Baskets.
- Dispersion Trades.
- Correlation Trades.

Trigger Redeemable Notes

- Four year structure with annual coupons of 9% until redemption or maturity.

- Annual observation dates with 100% redemption if the index is above initial level at observation date.

- Final payoff is 100% unless the index has in the interim dropped to below 50% of initial level, in which case the payoff is the minimum of 100% and the final value.

- The structure may be viewed as a bond less a down and in put with early redemption in an improving market.

- There is an upfront price paid, followed with coupons and 100% redemption except in a down market with loss of principal only with a strong market down move in the interim.

- The redemption may be postponed with redemption at $110\%, 120\%, 130\%$ only if the index is above initial value by these percentages, with final payoff in the absence of the down event allowing for participation in an ATM call on the market.

Vanilla Arithmetic and Product Cliquets

- Consider an example with one underlying index, the DJ Eurostoxx 50 with a notional of 100,000,000 Euros.

- The maturity is three years.

- There are 36 observation dates given by month ends till maturity.

- R_i is the return for month i defined by

$$R_i = \frac{S_i - S_{i-1}}{S_{i-1}}$$

where S_i is the closing price of the index on the i^{th} observation date.

- The local floor is the maximum loss taken each month while the gain is capped at the local cap. For a local floor of -10% and the local cap at 15% the locally floored and locally capped return each month is

$$U_i = Min(Max(-10\%, R_i), 15\%)$$

- The Arithmetic Cliquet pays subject to a global floor (GF), and cap (GC) the sum

$$Notional \times Min\left(Max\left(\sum_{i=1}^{36} U_i, GF\right), GC\right)$$

- The Product Cliquet pays subject to a global floor, and cap the sum

$$Notional \times Min\left(Max\left(\prod_{i=1}^{36} U_i, GF\right), GC\right)$$

Remarks on the Arithmetic and Product Vanilla Cliquet

- The basic product is to go long a periodic, here monthly, bull spread that incorporates the local floor and cap.

- However, a sufficiently large number of negative returns above the local cap may destroy earnings over the long term and hence the protection for the holder of the global floor.

- Similarly, a large number of positive returns below the local cap may amount to a large payout and hence the protection for the seller of the global cap.

- It is expected that in normal times the returns are between the local floor and local cap and there are no negative or positive trends to break the global floor and cap.

- Of course the floors and caps may be lifted with their associated costs and benefits.

- The global floors and caps introduce an additional level of convexity and concavity in the payoff above and beyond the local caps and floors.

The Reverse Cliquet

- The reverse compounding cliquet takes a position in the negative of the monthly return and the locally floored and capped return is now defined with a 13% cap and a -6% floor as

$$V_i = Min\,(Max(-R_i, -6\%), 13\%)$$

- This monthly payout may be summed or multiplied over a number of months and then capped and floored at a global level.

- The reverse arithmetic cliquet pays

$$Notional \times Min\left(Max\left(\sum_{i=1}^{36} V_i, GF\right), GC\right)$$

- The summation is replaced by the product for the product cliquet.

A Cancellable Reverse

- 4 annual observation dates

- underliers are 3 market indices, SPX, DJ Eurostoxx 50, Nikkei 225 Index.

- Early redemption condition

$$if\ for\ all\ i\ \frac{S_t^i}{S_0^i} > B_i$$

  where $B_i = 100\%, 95\%, 85\%, 75\%$ for $i = 1, 2, 3, 4$.

- Redemption payment at observation date t

$$Notional \times 6t\%$$

- Termination payment

$$Notional \times Max\left(0, 100\% - Min_{i=1,3}\left(\frac{S_4^i}{.75 S_0^i}\right)\right)$$

Swing Cliquet with Lock-In

- Notional in Quantoed Euro.
- 4 year maturity.
- 4 Annual Observation Dates.
- 20 underlying stocks with observation date prices $S_t^i, i = 1, \cdots, 20$; $t = 1, \cdots, 4$.
- Initial coupon $C_0 = 0$.

- Subsequent coupons $C_t, t = 1, \cdots, 4$

$$C_t = Max(C_{t-1}, MAD_t)$$

where

$$\begin{aligned} MAD_t &= Max(\left(Min_i(|R_t^i|) - x\%\right), 0) \\ R_t^i &= \frac{S_t^i - S_{t-1}^i}{S_{t-1}^i} \end{aligned}$$

- Locally the product pays if the minimum absolute return exceeds the strike of $x\%$.

- The coupons have a lock in determined by the previous coupon.

Reverse Swing Cliquet

- This product receives a cash flow that decreases with the absolute return and the coupon on a single underlier with a 15% local cap and a zero floor would have the structure

$$W_t = Max(.15 - |R_t|, 0)$$

- The arithmetic form would then sum these payouts subject to a global cap if necessary to protect against a large number of small returns.

Napoleonic Features

- DJ Eurostoxx 50 Napoleon Quarterly Equity Linked Note

- Quarterly observations on the Eurostoxx 50 with annual coupon consisting of 3% plus an equity amount determined as

$$5\% + \frac{1}{2} Minimum\ Quarterly\ Return$$
$$+ \frac{1}{2} Second\ Minimum\ Quarterly\ Return$$

  subject to a local floor of 0 and a local cap of 12%.

- The product is issued with a upfront price quoted like a bond. We may remove the 3% coupon component and then there is just an up front premium payment for the equity component. We may also get a quote in a swap format with the buyer paying Euribor plus a spread every three months till maturity.

Incorporating Lock Ins

- 100% principal protected note with profit lockins.

- 3 year maturity, 36 monthly observation dates with locally floored monthly returns

$$U_i = Min(R_i, 2.1\%)$$

- Payment at Maturity is

$$100\% Par + Max\left(0, Lockin, \sum_{i=1}^{36} U_i\right)$$

- The lockins are revised at each observation date as follows,

$$\begin{aligned} Lockin_0 &= 0 \\ if\ for\ any\ i \sum_{j=1}^{i} U_j &> 10\%, 20\%, 30\% \\ then\ Lockin_i &= .1, .2, .3 \end{aligned}$$

Intermediate Floors and Caps

- 5 Year Quarterly Capped, Yearly Range product.

- Option premium payed up front.

- Observation dates quarterly for 5 years. Denote by R_{yq} the return in quarter q of year y.

- The quarterly capped return is $U_{yq} = Min(R_{yq}, 5\%)$.

- The annual return is

$$V_y = \sum_{q=1}^{4} U_{yq}$$

- The payment at maturity is

$$Max\left(\left(\sum_{y=1}^{5} Min\left(Max(V_y, -5\%), 15\%\right) - X\%\right), 0\right)$$

- The quarterly capped returns are subjected to an annual floor of -5% and and annual cap of 15% and on the 5 year global return we have a call struck at $X\%$.

Melting Baskets

- 8 Year Autocancellable with Everest Basket on 25 names

- 8 annual observations on 25 stock prices.

- First two coupons $X\%$ to be determined.

- For the remaining years the coupon is

$$C_t = Max(0, 9\% + .3 Min_{i=1,N(t)} \left(\frac{S_t^i}{S_0^i} - 1 \right)$$

  if the swap has not been cancelled.

- At each observation date the worst performing stock is deleted from the basket if its price has fallen to below half its initial value.

- Swap is cancelled at observation date T when

$$\sum_{t=1}^{T} C_t = 20\%$$

- In this case the last coupon is truncated to meet the 20% condition.

Dispersion Trades 1

- 7 Year positive dispersion.

- 20 names in the basket, annual observation dates on 20 names with closing prices S_t^i.

- $PR = participation\ rate$, say 15%.

- Price is an up front option premium

- Initial coupon is zero. Thereafter it is

$$
\begin{aligned}
C_t &= PR \times \frac{1}{20} \sum_{i=1}^{20} Max\left(0, \frac{S_t^i - S_0^i}{S_0^i} - D_t\right) \\
D_t &= \frac{1}{20} \sum_{i=1}^{20} \frac{S_t^i - S_0^i}{S_0^i}
\end{aligned}
$$

Dispersion Trades 2

- 5 Year Auto Cancellable Dispersion Racer linked to Eurostoxx 50

- Counterparty pays 6 month Euribor flat actual/360.

- Annual observations on the DJ Eurostoxx 50 and 8 sector indices.

- We solve for $X\%$ where we have the cancellation provision if for some sector i at observation date t it is the case that

$$\frac{S_t^i}{S_{t-1}^i} - \frac{SX5E_t}{SX5E_{t-1}} \geq X\%$$

- Otherwise we pay

$$C_t = Max\left(.02, Max_i\left(\frac{S_t^i}{S_{t-1}^i} - \frac{SX5E_t}{SX5E_{t-1}}\right)\right) < X\%$$

- And if this value exceeds $X\%$ we cancel on the payout of $X\%$.

Correlation Products

- Daily observations on 20 shares for one year.

- Swap settlement with cash flow

$$Notional \times (\rho_{\mathsf{Re}\,alized} - \rho_{Strike})$$

- ρ Strike to be determined

- $\rho_{\mathsf{Re}\,alized}$ at maturity is determined as follows

$$\rho_{\mathsf{Re}\,alized} = \frac{2}{n(n-1)} \sum_{i=1,20,i<j} \rho_{ij}$$

- The pairwise correlations ρ_{ij} are statistical calculations of covariance of log returns divided by the product of the standard deviations.

Rationale for Products

- A basic principle in short horizon, say one to three month, derivative product design is the recognition that one may hold any function $c(S)$ of the stock price, accessing curvature by option positioning.

- The bond gives you a constant.

- The bond plus a stock gives you any line.

- When you add options to this mix then you can change slopes using calls struck at particular points and hence you can buy arbitrary functions of the stock price.

- The question then arises as to what function one should in fact purchase.

The Answer from Utility Theory

- The utility of a dollar in any state of the world, which here is the level of the stock price S one to three months hence, is proportional to the marginal utility of wealth in this state and the probability of this state.

- If we buy the function $c(S)$ the marginal utility is $U'(c(S))$ and the probability is $p(S)$ where $p(S)$ is the statistical probability of reaching the level S in one to three months.

- Hence the expected utility of a marginal dollar in the state defined by the stock price at S is $U'(c(S))p(S)$.

- One should clearly transfer money from states with a low expected utility of a marginal dollar, say S, to states with a high expected utility of a marginal dollar say S'. The trade is clearly beneficial.

- However, there are the terms of trade. The risk neutral probability that we recover from option prices, tells us the price of a dollar in the state the stock price is S. The number of dollars we can get in state S' for a dollar in state S is then given by these terms of trade $q(S)/q(S')$.

- Equating the marginal utilty lost to the marginal utility gained from such a trade we get the result

$$U'(c(S'))p(S')\frac{q(S)}{q(S')} = U'(c(S))p(S)$$

or the optimality principle enunciated below.

- For an optimal design $c^*(S)$ we must have that the expected marginal utility from a dollar in each state per initial dollar expended is constant across states or that

$$\frac{U'(c^*(S))p(S)}{q(S)} = \lambda$$

- It follows that the optimal product satisfies the condition

$$U'(c^*(S)) = \lambda \frac{q(S)}{p(S)}$$

- Now marginal utility is a decreasing function of wealth and we may associate with each level of marginal utility y the level of wealth that has this marginal utility, W by the function

$$W = I(y)$$

with the inverse function property

$$I(U'(W)) = W.$$

- Applying the marginal utility inverse function to both sides of our optimality condition yields the optimal product

$$c^*(S) = I\left(\lambda \frac{q(S)}{p(S)}\right)$$

- An instructive utility function in this context is log utility for which marginal utility is W^{-1} and

$$I(y) = \frac{1}{y}$$

- In this case

$$c^*(S) = \frac{1}{\lambda}\frac{p(S)}{q(S)}$$

- More generally the shape of the desired product follows the structure of

$$\frac{p(S)}{q(S)},$$

and importantly different levels of risk aversion impact the size but not the shape of the product. The shape can be marketed quite universally.

- Two basic principles may be extracted.

  - Buy access to cash when probability exceeds price and short or sell cash flows when price exceeds probability.

  - For marginal utilities going to infinity as wealth reaches a lower bound, avoid exposure to losses beyond this max loss level.

- Given some temporal stability of the two probability densities,
  - $p(S)$ the likelihood of reaching the level S in a few months, and
  - $q(S)$, the risk neutral probability or market price of a security paying a dollar contingent on this event

  one attains some stability in the optimal product defined by their ratio.

- A few examples illustrate the possibilities.

Log Gaussian Products

- The basic model is to consider both p, q to be log normal with $S(0) = 1$, common volatilities σ and mean returns μ and r respectively for the statistical and risk neutral case.

- The optimal product structure is then easily evaluated as

$$c^*(S) = S^{\frac{\mu - r}{\sigma^2}}$$

- Limiting the structure to an interval in the return, we get a cliquet product for $\mu > r$ and a reverse cliquet for $\mu < r$.

Consequences of diferences in volatilities

- Now consider differences in volatilities,
  - with statistical volatility below its risk neutral counterpart,
    - the tails have high price and low probability
      - and so we short the tails and buy cash flow in the neck of the distribution or a reverse swing cliquet.

- In the opposite case when statistical volatilities dominate, as may be relevant today, the neck gets pricier and the tails have more probability relative to price and we get the swing cliquet as the optimal product.

Estimated Products

- One may estimate both densities in the parametric class of say the $CGMY$ model, using time series data for the statistical density and options data for the risk neutral.

- The results for the year 2002 on the SPX, DAX, $FTSE$, $IBEX$, and $NIKKEI$ are

Statistical Estimation

	SPX	DAX	FTSE	IBEX	NIKKEI
vols	0.1679	0.2569	0.1718	0.2222	0.2445
C	13.02	23.04	.2927	2.79	5.11
G	94.64	65.24	51.99	63.08	68.57
M	100.2	78.10	56.37	75.60	66.16
Y	0.5348	0.4925	1.21	0.8963	0.7982

- The corresponding risk neutral results are, using averages for 12 mid month estimations,

Risk Neutral Parameter Estimates

	SPX	DAX	FTSE	IBEX	NIKKEI
vols	0.3332	0.5088	0.3246	0.3372	0.6182
C	0.8689	1.2594	0.2902	0.8757	3.6502
G	6.9420	5.7464	5.1308	8.2975	10.2088
M	31.1907	27.9887	41.7202	40.9873	28.5528
Y	0.8801	0.9914	1.1902	0.9625	0.9228

- For a one month horizon we present graphs for the statistical and risk neutral densities and the optimal product structure for a zero mean and interest rate.

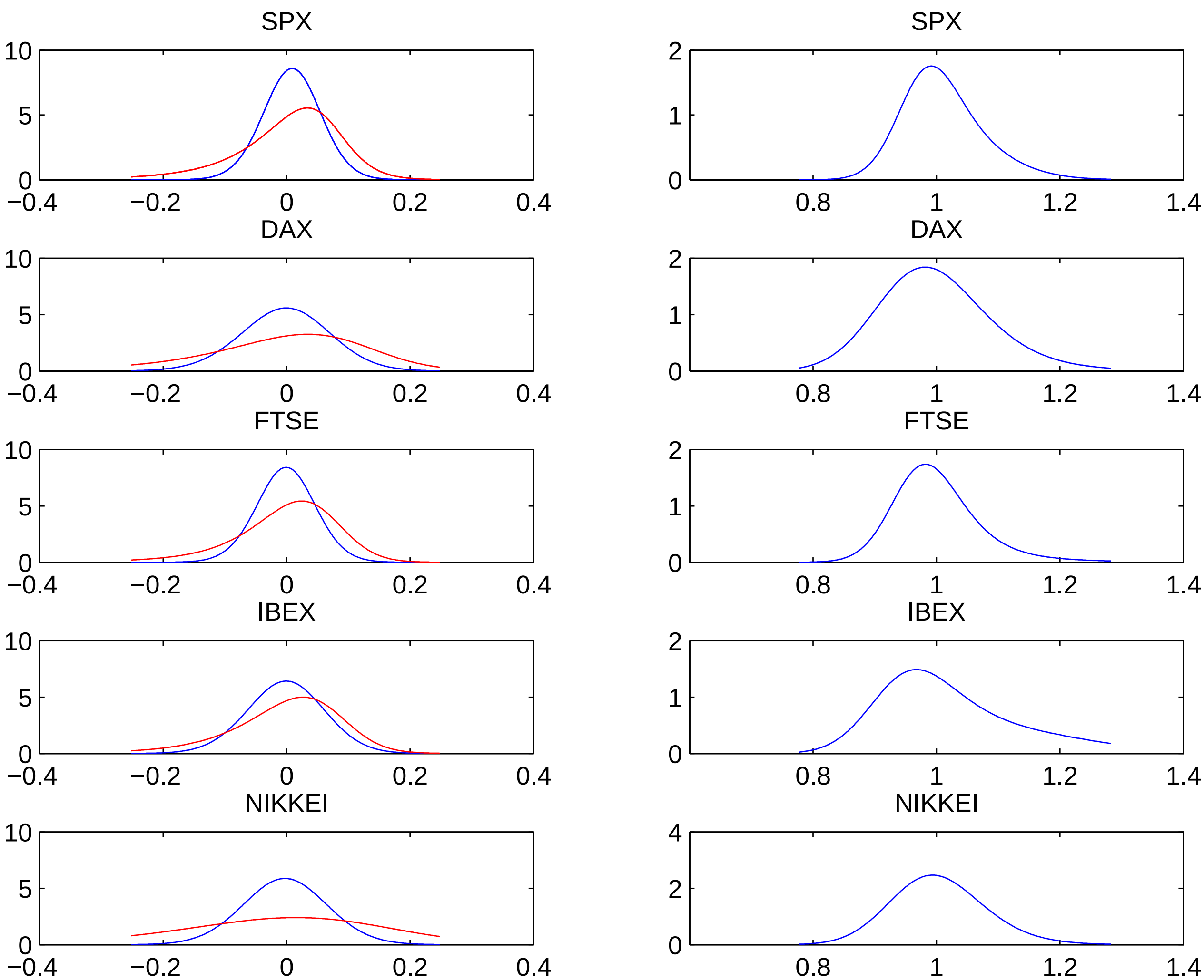

Figure 1:

Down and In Trigger Reedemable Notes

- Consider the down and in Trigger reedemable Note with a 50% down barrier. The Note pays 100 in the absence of the down event, and otherwise pays the minimum of the final value and 100.

- Using the above risk neutral and statistical parameters for $CGMY$ and methods for exact analytical pricing of equity default swaps for Lévy processes we determine the five year statistical and risk neutral probabilities of first passage to the 50% down barrier
  - at 2% statistically, and
  - 14% risk neutrally.

- The EDS quote at 50% recovery is
  - statistically $14bp$,and
  - risk neutrally $141bp$.
- The product clearly sells a high price low probability event without any exposure to negative cash flows.
- From the demand side this is a good product.
- To be a really good product, it must be one for the supply side as well. For this we need to understand its risks, how they are to hedged and managed.

Path Dependent Products

- In general for any prespecified set of underliers and observations dates we may let s denote the time path of these underliers on these dates for a particular scenario. In total we may contemplate some M scenarios indexed by $s = 1, \cdots, M$.

- For each scenario, each market participant can rank the scenario in terms of level of dissatisfaction with the outcome of his or her portfolio on this scenario, giving the highest score to the most dissatisfactory performance. Let R_s be this ranking of dissatisfaction with the particular scenario.

- The scenarios also have probabilites estimated by market participants at p_s for the scenario s.

- A product may be visualized as the vector $x = (x_s, s = 1, \cdots, M)$ that offers the present value cash flow x_s in scenario s.

- An estimate of the price w that a market participant is willing to pay for the product x can constructed as

$$w = \sum_s \frac{R_s p_s}{\left(\sum_s R_s p_s\right)} x_s$$

  a dissatisfaction times probability weighted average of the present value cash flows.

- If an investment house can effectively produce the product with cash flow x for an ask price

$$a(x) < w$$

  then there develops a healthy market for the structured product with cash flow x.

Correlation Trades

- When correlations rise portfolio volatilities rise leading to losses in short index option positions. For some market participants these scenarios of high correlation have a high dissatisfaction index. Products paying realized correlation are then attractive and could possibly be offered at reasonable prices as an insurance prior to the correlation event.