Financial Innovation and New Structured Products in the Equity World

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OUTLINE

- Risk exposures of products.
- Supply side of structured products.
- Modelling the bid and ask prices.

Describing the Risks

- The structured products are contingent claims on the paths of underliers and so they are exposed to movements in the prices of these underlying assets.

- They also embed many optionalities with respect to the paths of the underlying asset prices and can in principle be hedged by positions in options.

- They are therefore exposed to the movements in the option surface.

- They also deliver monies through time and this exposes their values to interest rate risk and the correlations between these and the underliers determining the cash flows.

- Many of these risks can be managed by positions in traded assets from time to time, as the exposure detected is deemed undesirable and needs to be mitigated.

- Of particular relevance in this context is products that build in what is called digital risk, where the asset position required to mitigate the risk goes to infinity and the risk can then not be realistically managed.

- Yet another important feature is nonlinearity or convexity/concavity of value which when sharp requires further optionalities to build a hedge or control the loss exposure.

Enumeration of Risks

- We may enumerate the risks as follows.

- From the underliers we have the
  - The delta with respect to each underlier.
  - The gamma with respect to each underlier in case of convexity or concavity.
  - The cross gamma or exposure to the second order cross partial with respect to two asset prices.

- From the option surface we have
  - The level of volatilities
  - The level of skews measured for instance as the difference between the $90 - 110$ implied volatilities.
  - The convexity of the implied volatility curve.
  - The volatility spread between longer and shorter maturity options.

- The gammas with respect to each of the above option surface risks in case of sharp convexities or concavities.
  * This leads to the
    · volgamma,
    · skewgamma,
    · convexitygamma, and
    · the volspreadgamma.

- From the fixed income arena
  - Exposure to the yield curve.
  - Exposure to correlations between the underliers and interest rates.

Assessing the Risks

- We may value the product using a model calibrated to the relatively more liquid surface of option prices and then we may shift the surface to assess for example the surface risks.

- This is a complex calculation that is difficult to do for each product and each risk element.

- Somewhat cruder but helpful, is to understand the risks in the components of the products, that is the risk in each of the promised coupons with their built in optionalities.

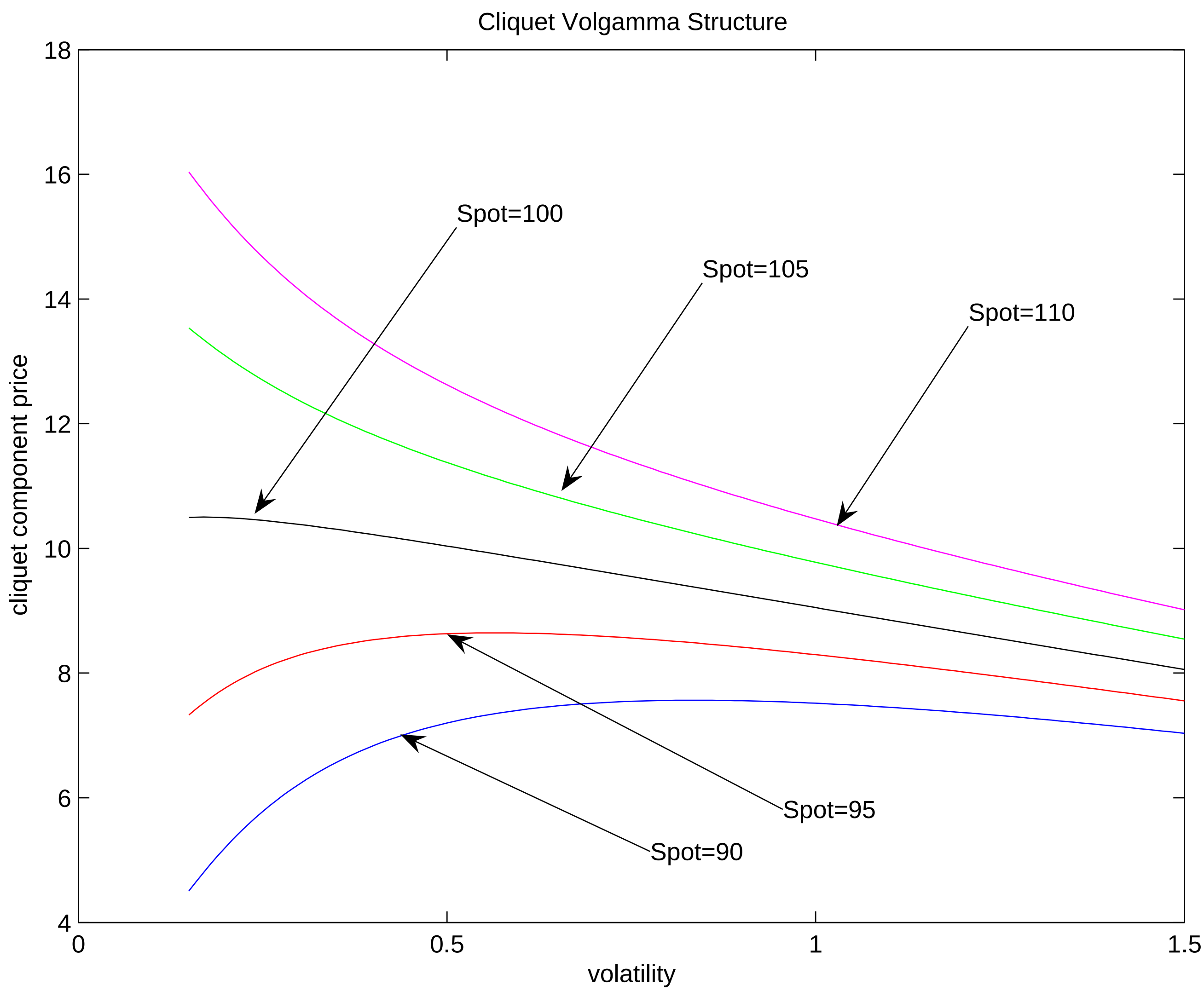
- For particular products we consider the exposure to volgamma, skewgamma and the convexity gamma.

Surface Exposures of Products

- We present with a view to illustrating option surface deltas and gammas the cliquet and reverse cliquet volgamma and the ATM swing cliquet skewgamma.

Remarks on Cliquet Volgamma

- For at the money and for large volatilities the volgamma is relatively damped though we have a voldelta.
  - In this region the vol risk could be managed exposure to volatility via ATM options.
- For lower volatilities, when the spot has risen the volgamma is positive and stochasticity in volatility would raise values and is a risk for the short cliquet position.
- For lower volatilities, when the spot has fallen the volgamma is negative and stochasticity in volatility is a risk for the long cliquet position.
- These considerations lead to demands for evaluating the product using stochastic volatility models in such regions.



Cliquet Volgamma Structure
18
16
14
12
10
8
6
4
0
0.5
1
1.5
volatility
cliquet component price
Spot=100
Spot=105
Spot=110
Spot=95
Spot=90

Figure 1:

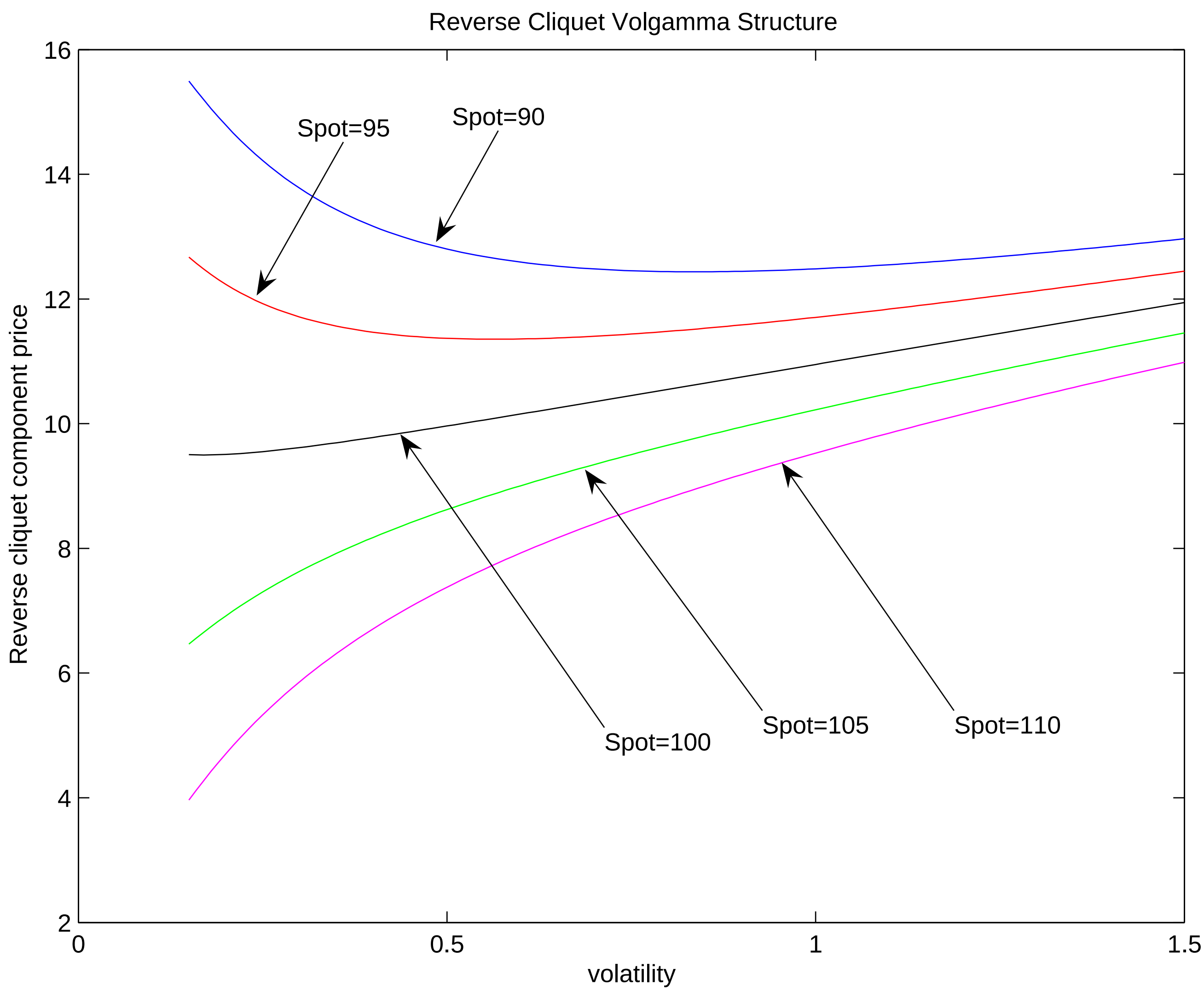
- Additionally it is anticipated that option hedges for volatility may need to be rebalanced and the costs for these must be anticipated.

Remarks on Reverse Cliquet Volgamma

- Again for at the money and for large volatilities the volgamma is relatively damped though we have a voldelta.
  - In this region the vol risk could be managed exposure to volatility via ATM options.

- For lower volatilities, when the spot has risen the volgamma is negative and stochasticity in volatility is a risk for the long cliquet position.

- For lower volatilities, when the spot has fallen the volgamma is positive and stochasticity in volatility is a risk for the short cliquet position.



Reverse Cliquet Volgamma Structure
16
14
12
10
8
6
4
2
0
0.5
1
1.5
volatility
Reverse cliquet component price
Spot=95
Spot=90
Spot=100
Spot=105
Spot=110

Figure 2:

Remarks of Swing Cliquet Skewgamma

- The presence of convexity in the value of the swing cliquet with respect to the skew suggests that stochasticity in the skew may impact the value of this claim upwards.

- Valuation of such claims under a stochastic skew model is then of some interest.

- Models of this type are now under construction.

Marked to Market Risks of the Variance Swap

- The variance swap contract trades quite popularly and options on volatility are beginning to trade. The risk exposures of these contracts are of interest as they are also used for covariance swap contract construction.

- The well known robust hedge for the variance swap is the short position in the log contract.

- This may easily be priced by any model fitting the option smile at a single maturity with a known characteristic function for the log price relative.

- We may also extract from the market the ATM volatility, the 95-105 3 month skew and the 95-100-105 3 month convexity.

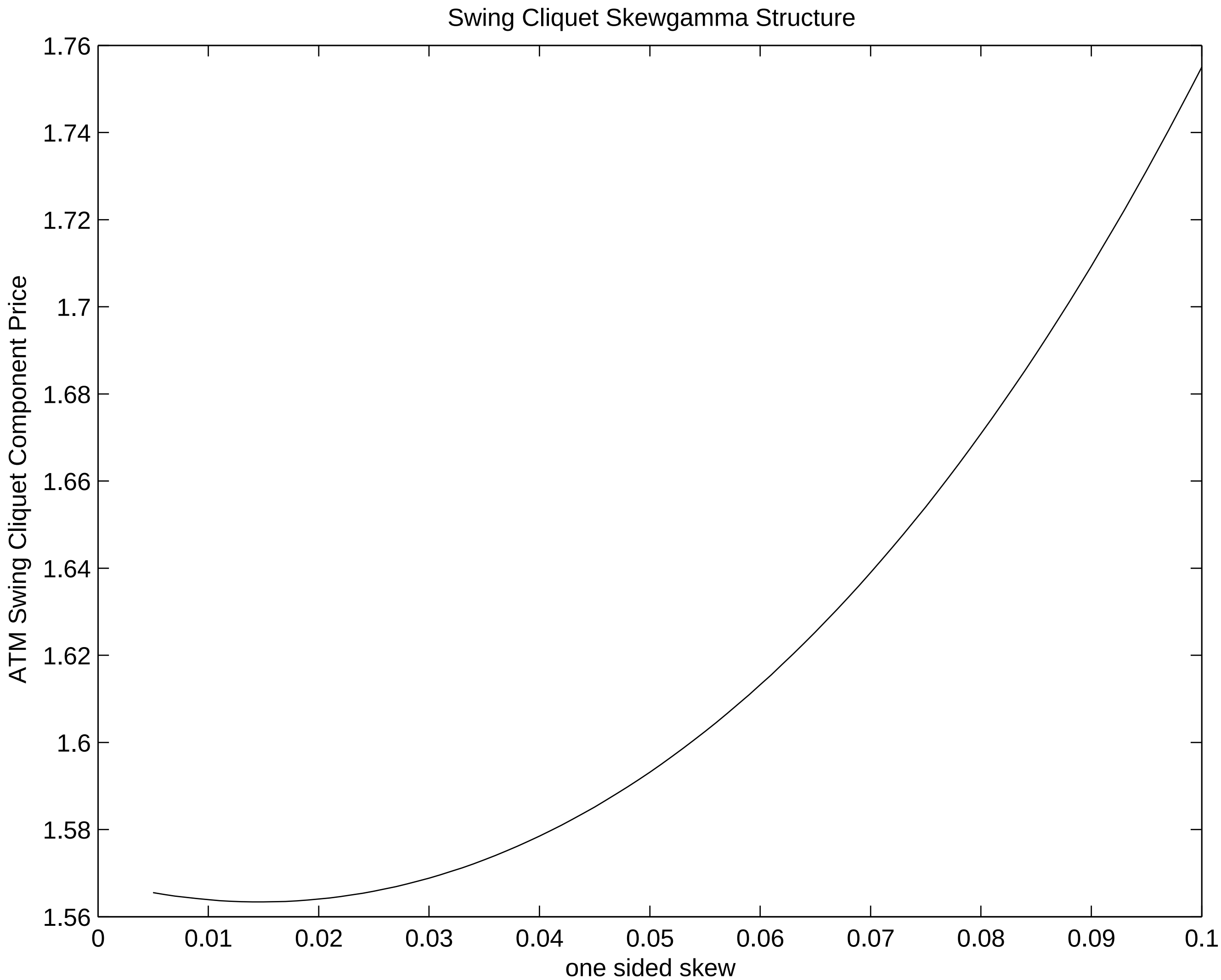

Swing Cliquet Skewgamma Structure
ATM Swing Cliquet Component Price
1.76
1.74
1.72
1.7
1.68
1.66
1.64
1.62
1.6
1.58
1.56
0
0.01
0.02
0.03
0.04
0.05
0.06
0.07
0.08
0.09
0.1
one sided skew

Figure 3:

Skew gamma in variance swaps

- The presence of a skew gamma in the variance swap contract is yet another reason for models with stochastic skews.

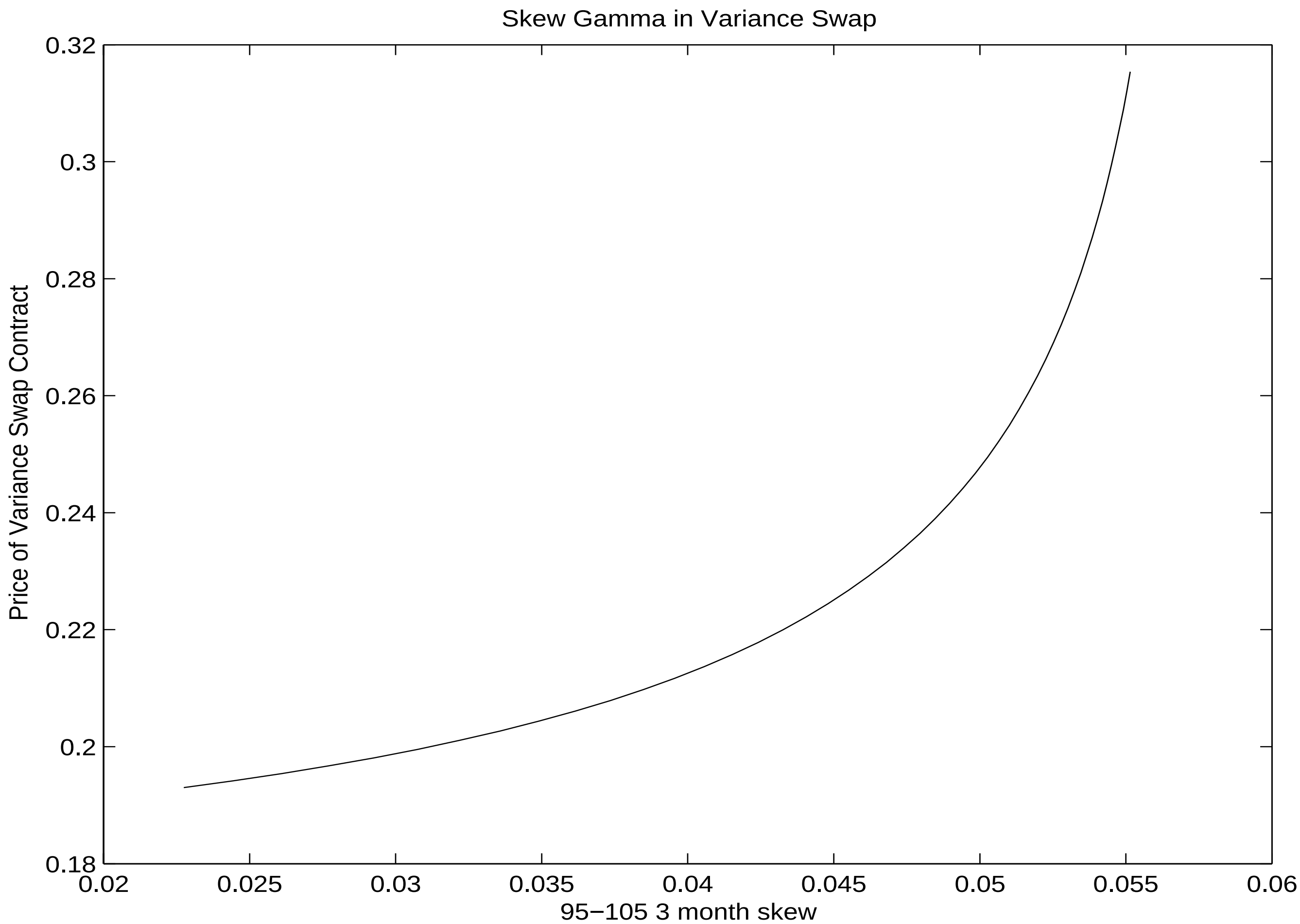
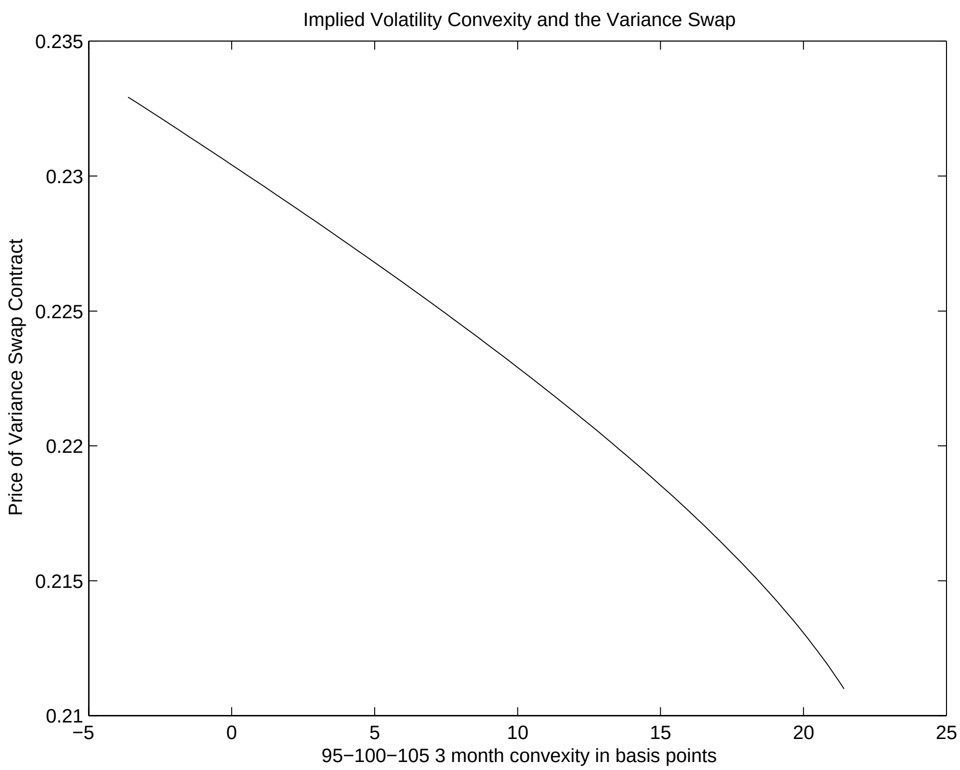

Skew Gamma in Variance Swap
Price of Variance Swap Contract
95−105 3 month skew
0.32
0.3
0.28
0.26
0.24
0.22
0.2
0.18
0.02
0.025
0.03
0.035
0.04
0.045
0.05
0.055
0.06

Figure 4:



Implied Volatility Convexity and the Variance Swap
0.235
0.23
0.225
0.22
0.215
0.21
Price of Variance Swap Contract
−5
0
5
10
15
20
25
95−100−105 3 month convexity in basis points

Figure 5:

Importance of Forward Skews

- Products with substantial skew gammas also have significant skew deltas and their values are influenced by movements in the skews.

- The model values of these products are subject to the movements of skews in the models.

- One of the popularly used models for managing and controlling structured product risks is the local volatility model.

- Unfortunately, as the following graph shows, the behavior of forward volatilities and skews leaves much to be desired in these models, making them questionable for use in the management of especially cliquet structures.

- These considerations led to the development of local Levy models that better manage forward skew evolution.

Cross Gamma Effects

- Structures involving Napoleonic features or dependencies on worst performers as the minimum price across a set of names have a delta that is primarily determined by the names that are currently the worst performer.

- When another name replaces these names as the worst performer, the primary delta effect switches to the new asset.

- As a result, the delta with respect to the previous worst performer can drop sharply without any movement in its price but simply by the drop in the value of the new worst performer.

- These effects are called cross gamma effects and for certain products one needs to watch out for these movements.

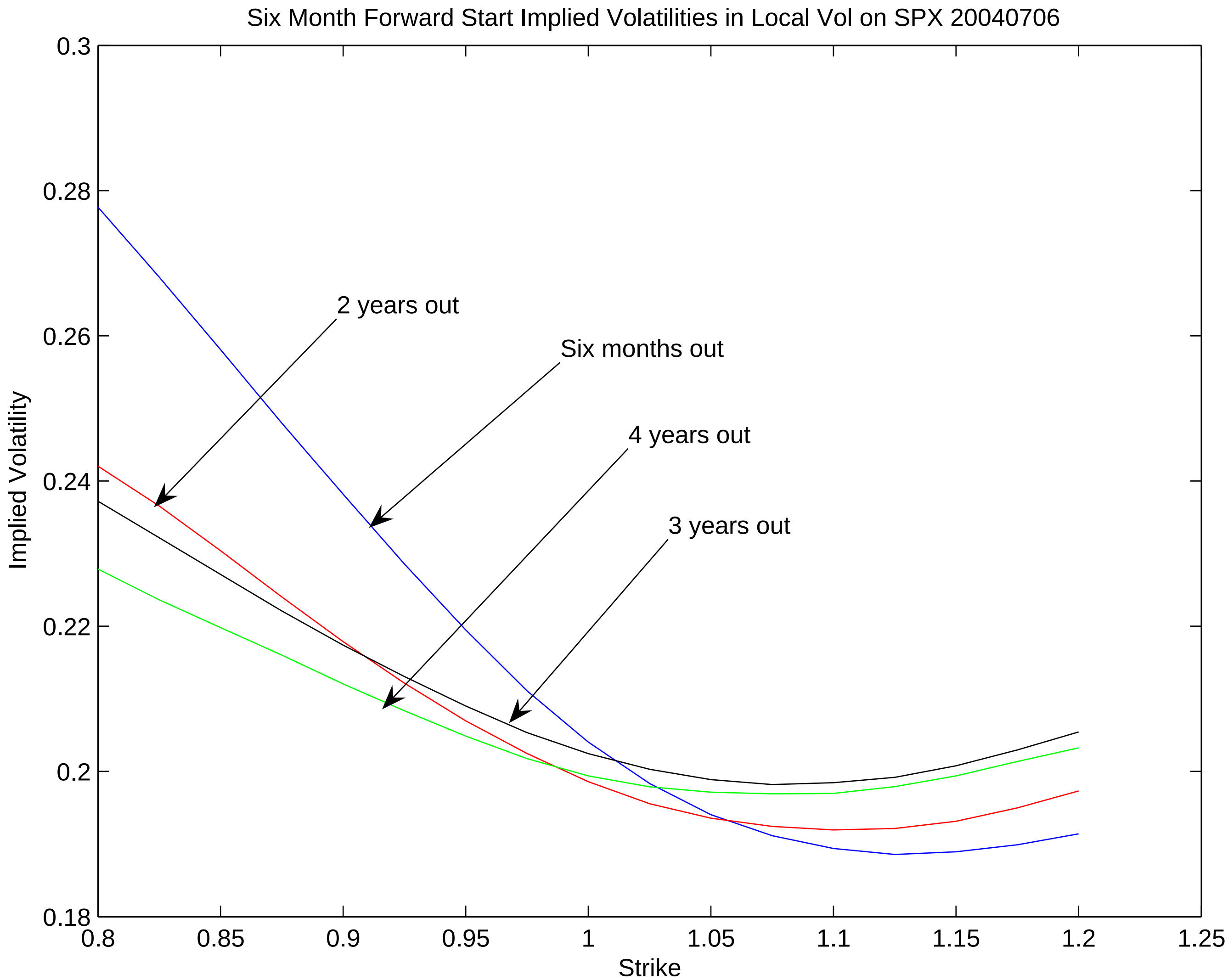

Six Month Forward Start Implied Volatilities in Local Vol on SPX 20040706
Implied Volatility
Strike
2 years out
Six months out
4 years out
3 years out
0.3
0.28
0.26
0.24
0.22
0.2
0.18
0.8
0.85
0.9
0.95
1
1.05
1.1
1.15
1.2
1.25

Figure 6:

Cross Gamma in Basket Option Trades

- Structured Products provided optionality over a basket of names may often be hedged by positions in the individual options. The result is a value function that depends nonlinearly on both the asset values.

- One may evaluate across the state space of asset prices, the exposure of the delta hedged value function to discover saddle point structures whereby we have zero delta and positive gamma in each direction so that the position looks good.

- However there may be a strong negative cross gamma with the consequence that if a pair of prices moves in a certain direction then significant losses are to be incurred.

- An eigen value analysis of the matrix of second derivatives will reveal all the negative and positive gamma directions.

- The hedged structure needs to be watchful of these directional gammas.

Structures Exposed to Digital Risk

- Sharp barrier clauses like knock outs or knock ins of value on specific events lead to difficulties in managing the risk near the barrier.

- It is best to cover these situations by option positioning across the barrier event, as opposed to relying on the delta hedging mechanisms.

- The positioning should be undertaken some what in advance of the barrier to control the cost of insurance purchase.

Correlation between Equity and Interest Rate Risks

- Equity Returns and interest rate movements go through periods of differing levels of correlation.

- A period of strong positive correlation has equity linked notes making healthy payouts in higher interest markets and this raises the value of the payout to the investor.

- One may therefore wish to assess the value of the contract using a model that allowed for positive correlation between equity returns and interest rates.

- Typically, if the cash flows are more bond like with their volatility diminished, the correlation of the timing with the interest rate environment gets relatively more important.

- To the extent that equity volatility feeds into the payout pattern the correlation effects with interest rate movements get dominated.

- Particulary suited to such investigations are factor models that are jointly driving the equity and the instantaneous short rate.

Supply Side of Structured Products

- There are many risks to be understood, managed and priced into the ask price of structured products.

- How can we go about the production of liabilities incurred on the sale of a structured product.

- Clearly, to the extent we can take positions in current and future option market assets to cover the liability, we have some understanding of how to produce the structured product and what to charge for it.

- Exact replication is however, unlikely and we need to determine how one may make the holding of the residual an acceptable risk.

- These ideas lead to a stylised model for the determination of structured product ask, and if necessary bid, prices.

The Relatively Liquid Hedging Assets

- We go back to viewing the structured product as a scenario contingent vector of payouts $x = (x_s, s = 1, \cdots, M)$.

- Next we introduce traded assets whose prices are known along the scenario paths and by financing their prices we generate a matrix of zero cost cash flows Y where Y_{js} is the present value cash flow accessed by positioning in zero cost liquid asset j on scenario s.

- These assets could include the financed purchase of stock for some interval of time as well as the financed purchase of current or forward starting options held to maturity.

- If we adopt the hedge that takes the position α_j in liquid asset j then our residual cash flow is

$$\alpha' Y - x'$$

- If this position is zero or nonnegative, it is clearly acceptable.

- However, this is not likely to be the case.

Acceptable Risks

- Acceptable Risks have been effectively defined as a convex cone containing the positive orthant.

- Intuitively, if a sufficient number of counterparties value the gains in excess of the losses, then the risk is acceptable.

- For practical purposes this amounts to the existence of valuations that are probability times path dissatisfaction weighted averages of the risky cash flows, all which must be positive for a risk to be accepted.

- Let B be the matrix of such valuation measures used for testing acceptability. (See Carr, Geman, Madan JFE 2002 for greater details).

- For the risk to be acceptable we must have

$$(\alpha' Y - x')B \geq 0$$

- It may be the case that such is not possible and so we add cash to the position, that is essentially the price sought for the sale of the liability x.

- The smallest number a that renders

$$a + \left(\alpha' Y - x'\right) B \geq 0$$

is an admissible price for the sale of x coupled with hedge α.

The Ask Price Problem

- The Ask price problem is to find $a(x)$ such that

$$
\begin{aligned}
a(x) &= Min_{a,\alpha}\ a \\
&\quad S.T. \left(x' - \alpha' Y\right) B \leq a
\end{aligned}
$$

- The ask price is the smallest value needed to cover all the valuation shortfalls net of the hedge.

- By virtue of being a minimization problem defined with respect to a linear constraint set defined by x it is clear that $a(x)$ will be a convex functional of the cash flows x and linear pricing does not hold.

- Arbitrage is however, nonetheless excluded by virtue of bid ask spreads.

The Bid Price Problem

- If the cash flow x is to be bought rather than sold, then the money to pay for it is to be raised by a finding zero cost hedge that makes the residual less the price a, acceptable or

$$(x' - \alpha' Y)B - a \geq 0$$

- The bid price problem is to find $b(x)$ such that

$$\begin{aligned} b(x) &= Max_{a,\alpha} a \\ & S.T. \left(x' - \alpha' Y \right) B \geq a \end{aligned}$$

- The bid price is the largest number we may extract from all valuations net of a hedge.

- By virtue of being a maximization problem defined with respect to a linear constraint set defined by x, the function $b(x)$ is concave in the cash flows x.

Implications of the Dual Problems

- These ask and bid price problems having informative dual problems that define

$$
\begin{aligned}
a(x) \;&=\; Max_q \; x'Bq \\
&\quad S.T. \; YBq = 0 \\
&\quad \mathbf{1}^T q = 1
\end{aligned}
$$

- The ask price is the maximum price over a set of models that calibrate the liquid prices.

- As well as

$$
\begin{aligned}
b(x) \;&=\; Min_q \; x'Bq \\
&\quad S.T. \; YBq = 0 \\
&\quad \mathbf{1}^T q = 1
\end{aligned}
$$

- The bid price is the minimum price over a set models that calibrate the liquid prices.

- From which we learn immediately that $b(x) < a(x)$.

- The profit from buying a package $1/2x_1 + 1/2x_2$ and selling the pieces is then

$$\begin{aligned} & -a(\frac{1}{2}x_1 + \frac{1}{2}x_2) + \frac{1}{2}b(x_1) + \frac{1}{2}b(x_2) \\ \leq \; & -a(\frac{1}{2}x_1 + \frac{1}{2}x_2) + b\left(\frac{1}{2}x_1 + \frac{1}{2}x_2\right) < 0 \end{aligned}$$

- Similarly the profit from selling the package $1/2x_1 + 1/2x_2$ and buying the pieces is

$$\begin{aligned} & b(\frac{1}{2}x_1 + \frac{1}{2}x_2) - \frac{1}{2}a(x_1) - \frac{1}{2}a(x_2) \\ \leq \; & b(\frac{1}{2}x_1 + \frac{1}{2}x_2) - a\left(\frac{1}{2}x_1 + \frac{1}{2}x_2\right) < 0 \end{aligned}$$

Acceptability, Hedging, and Arbitrage

- Given a uniform definition or agreement on the acceptability of risks, or agreement on the measures that test acceptability, one may define acceptable hedges that yield concave ask prices and convex bid prices.
  - Importantly, this structure excludes market participants from exposure to arbitrage by counterparties.

- Additionally, the differences generally observed in the prices of structured products using vanilla option calibrated models are no evidence of model risk, as none of these prices is a price.
  - Their maximum is an ask price while their minimum is a bid price.

- Furthermore, it is a natural consequence of the technology of hedging to acceptability that different products are priced using different models.
  - This is not an inconsistency, but just the recognition that the ask price for different products, seen as the maximum price over a set of calibrated models, attains this maximum at different models.
- Of course the fewer the set of risks that are acceptable, the wider is the bid ask spread and the less likely that there is a trading counterparty for any particular product.
  - Consensus over acceptability is attained as the broad structure of models in use is eventually widely disseminated.