Sato Processes and the Valuation of Structured Products

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Financial Innovation Workshop
7 City Learning
London
June 28

Outline

- The fast developing market for Equity Structured Products
- Pricing and Hedging Principles Relevant for Structured Products
- Some Classical Models
- Introduction to Sato Processes
- Comparison of forward and spot Sato returns
- Realized Variance Options and Sato Processes
- Unified Simulation of Sato Processes by Ziggurat method
- Classical Model Details and Calibration Results
- Structured Products Priced
- Pricing Results
- Conclusion

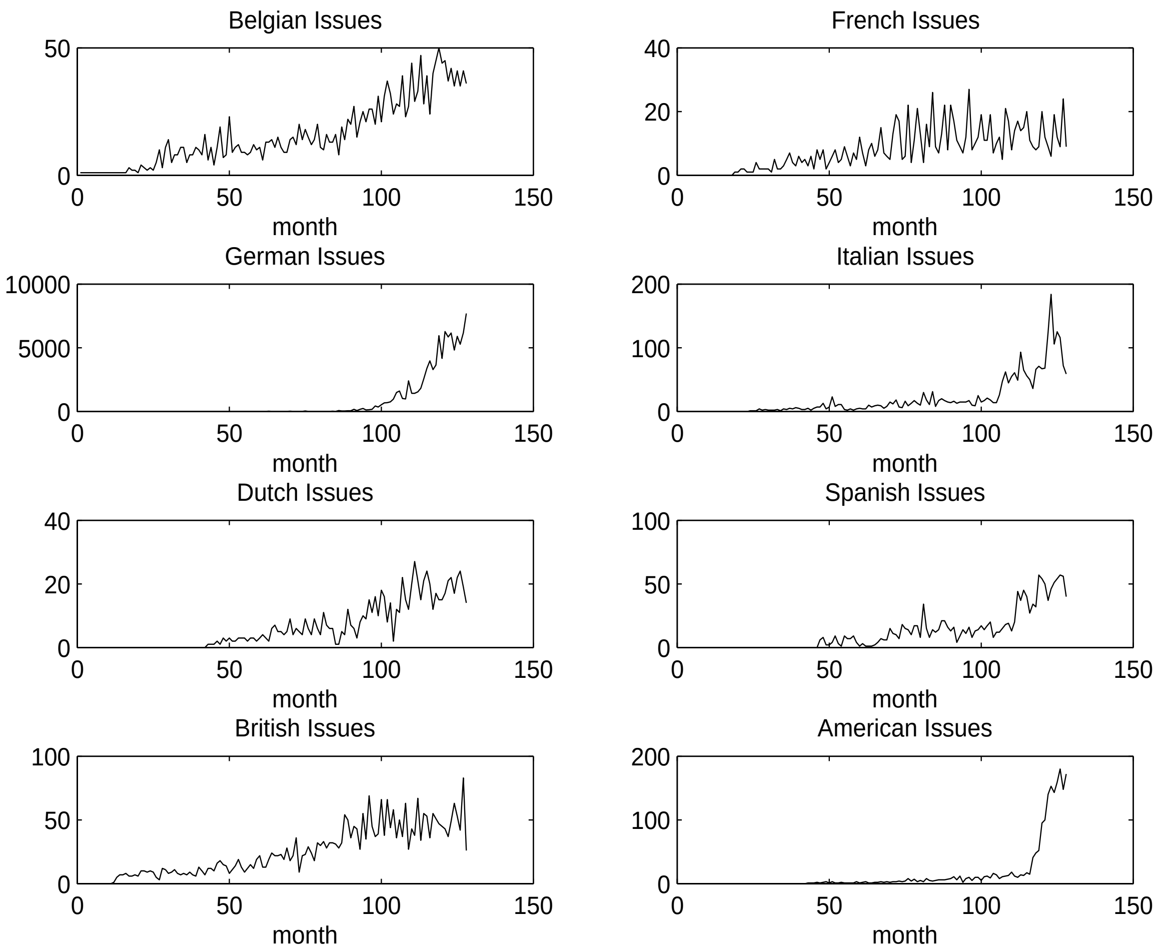
Market for Equity Structured Products

- Equity Structured Products have cash flows defined as functions of the path of Stock prices,
  - from contract initiation
  - to maturity or early termination
- Examples include
  - a host locally floored or capped and globally floored or capped arithmetic or product cliquets
  - options on realized variance
  - swing and reverse swing cliquets.

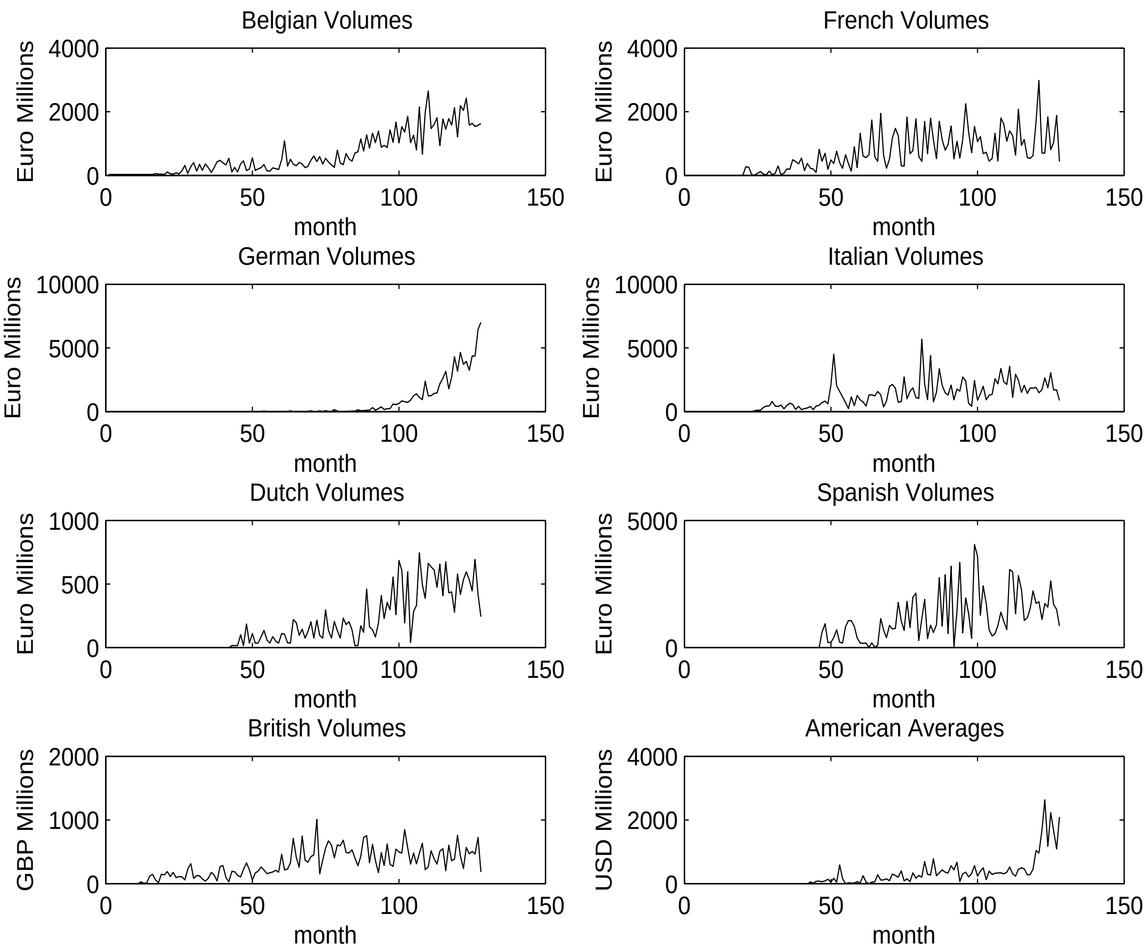
- There are over 140,000 product offerings by 900 companies with a total world sales of 600 billion Euros
- Fastest growing investment class in the US with 2004 notional at 12 billion rising to 50 billion in 2005 and growing.
- We present graphs of issues and volumes in 8 world financial markets.
  - Issues
    - Steady growth in issues in Belgium, Netherlands, Spain, UK
    - Stabilisation in France
    - Rapid Growth in Germany, Italy and US.
  - Volumes
    - Steady growth in most markets
    - Rapid Growth in Germany and US.

Pricing and Hedging Principles

- The first and foremost principle to be understood is where risk neutral pricing is relevant and why for structured products risk neutral pricing is not relevant.
- The critical principle underlying risk neutral pricing is the idea of pricing all products under a single measure or change of measure.
- The main motivation is linearity of the pricing operator backed by the recognition that in the absence of such a linearity there is a simple arbitrage, buy or sell the component cash flows A, B and sell or buy the package $(A + B)$.


Belgian Issues
French Issues
German Issues
Italian Issues
Dutch Issues
Spanish Issues
British Issues
American Issues
month

1.Issues


Belgian Volumes
French Volumes
German Volumes
Italian Volumes
Dutch Volumes
Spanish Volumes
British Volumes
American Averages
Euro Millions
GBP Millions
USD Millions
month
0
50
100
150
4000
2000
10000
5000
1000
500

2.Volumes

- This argument requires trading in both directions at the same price.
- For structured products there is little product standardization, few secondary markets, and buying is at an ask price with sales at the bid and these are widely different.
- Bidirectional trading at some midquote is mythical.

The Relatively Liquid Hedging Assets

- We can view the structured product as a scenario or path contingent vector of total present value payouts $x = (x_s, s = 1, \cdots, M)$.
- Next we introduce traded assets whose prices are known along the scenario paths and by financing the trades we generate a matrix of zero cost cash flows Y where Y_{js} is the present value cash flow, accessed by zero cost positioning, in liquid asset j on scenario s.
- These assets could include the financed purchase of stock for some interval of time as well as the financed purchase of options held for a future period.
- We are ignoring dynamic considerations as our points can be made in this simpler context and remain valid in the the more realistic dynamic setting.

Acceptable Risks

- If we adopt the hedge that takes the position α_j in liquid asset j then our residual cash flow is
$$\alpha' Y - x'$$
- If this position is zero or nonnegative, it is clearly acceptable.
- More Generally Acceptable Risks have been effectively defined as a convex cone containing the positive orthant.
- Intuitively, if a sufficient number of counterparties value the gains in excess of the losses, then the risk is acceptable.
- For practical purposes this amounts to the existence of valuations that are probability times path dissatisfaction weighted averages of the risky cash flows, all which must be positive for a risk to be accepted.

- Let B be the matrix of such valuation measures used for testing acceptability. (See Carr, Geman, Madan JFE 2002 for greater details).
- For the risk to be acceptable we must have
$$(\alpha' Y - x')B \geq 0$$
- It may be the case that such is not possible and so we add cash to the position, that is essentially the price sought for the sale of the liability x.
- The smallest number a that renders
$$a + (\alpha' Y - x')\, B \geq 0$$
is an admissible price for the sale of x coupled with hedge α.

The Ask Price Problem

- The Ask price problem is to find $a(x)$ such that

$$\begin{aligned} a(x) &= Min_{a,\alpha}\, a \\ &\quad S.T.\, (x' - \alpha' Y)\, B \le a \end{aligned}$$

- The ask price is the smallest value needed to cover all the valuation shortfalls net of the hedge.
- By virtue of being a minimization problem defined with respect to a linear constraint set defined by x it is clear that $a(x)$ will be a **convex** functional of the cash flows x and linear or risk neutral pricing does not hold.

The Bid Price Problem

- If the cash flow x is to be bought rather than sold, then the money to pay for it is to be raised by a finding zero cost hedge that makes the residual less the price a, acceptable or
$$(x' - \alpha' Y)B - a \geq 0$$
- The bid price problem is to find $b(x)$ such that
$$\begin{aligned} b(x) &= Max_{a,\alpha} a \\ &\quad S.T. \, (x' - \alpha' Y) \, B \geq a \end{aligned}$$
- The bid price is the largest number we may extract from all valuations net of a hedge.
- By virtue of being a maximization problem defined with respect to a linear constraint set defined by x, the function $b(x)$ is **concave** in the cash flows x and again linear or risk neutral pricing does not hold.

Implications of the Dual Problems

- These ask and bid price problems having informative dual problems that define

$$
\begin{aligned}
a(x) \;=\; & Max_q \, x'Bq \\
& S.T. \, YBq = 0 \\
& \mathbf{1}^T q = 1
\end{aligned}
$$

- The ask price is the maximum price over a set of models that calibrate the liquid prices.
- As well as

$$
\begin{aligned}
b(x) \;=\; & Min_q \, x'Bq \\
& S.T. \, YBq = 0 \\
& \mathbf{1}^T q = 1
\end{aligned}
$$

- The bid price is the minimum price over a set models that calibrate the liquid prices.

- Risk neutral models are to be used to calibrate the liquid asset prices and the sup over such models is the structured product ask price while the infimum is the bid price.
- All other numbers are just numbers and not prices and are not to be seen as evidence of model risk.
- The real risks lie in the definition of the cone of acceptability.

Nonlinear Pricing and Absence of Arbitrage

- For a uniform cone of acceptability across market participants
- We learn immediately that $b(x) < a(x)$.
- Furthermore, the profit from buying a package $1/2x_1 + 1/2x_2$ and selling the pieces is then

$$\begin{aligned} & -a(\frac{1}{2}x_1 + \frac{1}{2}x_2) + \frac{1}{2}b(x_1) + \frac{1}{2}b(x_2) \\ \leq \; & -a(\frac{1}{2}x_1 + \frac{1}{2}x_2) + b\left(\frac{1}{2}x_1 + \frac{1}{2}x_2\right) < 0 \end{aligned}$$

- Similarly the profit from selling the package $1/2x_1 + 1/2x_2$ and buying the pieces is

$$\begin{aligned} & b(\frac{1}{2}x_1 + \frac{1}{2}x_2) - \frac{1}{2}a(x_1) - \frac{1}{2}a(x_2) \\ \leq \; & b(\frac{1}{2}x_1 + \frac{1}{2}x_2) - a\left(\frac{1}{2}x_1 + \frac{1}{2}x_2\right) < 0 \end{aligned}$$

Lessons on Acceptability, Hedging, and Arbitrage

- Given a uniform definition or agreement on the acceptability of risks
  - Market participants are excluded from exposure to arbitrage by counterparties.
- Additionally, the differences generally observed in the prices of structured products using vanilla option calibrated models are no evidence of model risk, as none of these prices is a price.
  - Their maximum is an ask price while their minimum is a bid price.
- It is a natural consequence of hedging to acceptability that different products are priced using different models.
  - This is not an inconsistency, but the consequence of noting that the maximum price over a set of calibrated models, occurs for different products at different models.

Classical Models

- Apart from the underlying asset the relatively most liquid traded assets with market quotes that may be used for hedging are the vanilla options on the underlying.
- The most classical model is the Black-Merton-Scholes geometric Brownian motion model.
  - Here we have a total failure in synthesizing the vanilla option surface in any dimension of either strike or maturity as evidenced by the strike smile and the term structure of implied volatilities at a given level of moneyness.
    * Consequently this model is never used for the analysis of a structured product.

- The next advance was the use of Lévy models like the variance gamma, the hyperbolic model and their generalizations.
  - These models can capture the strike variation for a fixed maturity.
  - On occasion, they have also captured the term structure for a fixed strike.
    - They are on occasion therefore employed for products with a dominating cash flow at a given time point.

– It is well known that Lévy models fail to capture the joint surface of option prices across maturity and strike
  * They have a theoretical term structure in skewness and excess kurtosis inconsistent with market prices
    · with skewness falling like the square root of maturity
    · excess kurtosis falling like maturity
    · both of which are constant or rising in markets.
  * As a consequence they are not used for products with balanced cash flows spread out over multiple dates.

The Models that are used

- Models that match the surface of option prices well enough to be considered include
  - Heston Stochastic Volatility, (HSV) for longer dated cash flows, as it fails at the front end.
  - Stochastic Volatility with Compound Gaussian Jumps (SVJ).
  - Lévy models with stochastic volatility, $(VGSA, NIGSA, CGMYSA)$.
  - Local Volatility (LV). All skew from leverage function.
  - Local Lévy models (LL). Some skew hard wired into skewed local motion.

Introduction to Sato Processes

The Use of Limit Laws for the unit time distribution

- A classical motivation for using the Gaussian model is that it is a limit law and over any substantial period there are many independent effects on the stock price. This is often appealed to in elementary classes presenting the Gaussian model for the first time.
- Limit laws have been studied as far back as Lévy (1937) and Khintchine (1938) and the class of such laws, once one allows for arbitrary scaling and centering coefficients, are the self decomposable laws.

- A probability law of a random variable X is said to self decomposable just if for every constant c, $0 < c < 1$ there exists an independent random variable $X^{(c)}$ such that
$$X \stackrel{law}{=} cX + X^{(c)}.$$
- From a financial perspective this an important class of random variable models for the unit time distribution as independent effects on the return may need to be scaled to be brought to comparable orders of magnitude before scaling by the square root of n becomes relevant. Such considerations motivate arbitrary scaling factors and point to self decomposable laws as candidate models.

- Self decomposable laws are infinitely divisible and may be characterized nicely in terms of the Lévy density that must have the form
$$\frac{h(x)}{|x|}\mathbf{1}_{x<0} + \frac{h(x)}{x}\mathbf{1}_{x>0}$$
where h is increasing for negative x and decreasing for positive x.
- We call the function $h(x)$ the self decomposability characteristic.
- We note importantly that many finite activity models, the "so-called" jump diffusion models of the literature employing either Laplace or Gaussian jump size distributions are not self decomposable laws in their jump component.

Processes associated with self decomposable laws at unit time

- Given a candidate risk neutral self decomposable law at unit time we consider risk neutral laws at other time points defined by the scaling property. Specifically we consider defining a process $Y(t)$ with the property
$$Y(\lambda t) \stackrel{(d)}{=} a(\lambda)Y(t).$$
- It is easily seen on applying the above property to $\lambda\mu$ in two ways that we must have
$$a(t) = t^{\gamma}.$$

- Sato shows that one may construct an additive (inhomogeneous independent increment), self similar processes that matches at unit time a prespecified self decomposable law.
- The inhomogeneous Lévy density for the additive process may be explicitly identified in terms of the self decomposability characteristic and is given by $g(y,t)$ where

$$\begin{aligned} g(y,t) &= -\frac{\gamma h'\left(\frac{y}{t^{\gamma}}\right)}{t^{1+\gamma}}\mathbf{1}_{y<0} \\ &\quad +\frac{\gamma h'\left(\frac{y}{t^{\gamma}}\right)}{t^{1+\gamma}}\mathbf{1}_{y>0} \end{aligned}$$

- Note on making the change of variable

$$u = \frac{y}{t^{\gamma}}$$

and writing

$$\begin{aligned} g(y,t)dydt &= -\gamma h'(u)\,\mathbf{1}_{u>0}du\; dlogt \\ &\quad +\gamma h'(u)\,\mathbf{1}_{u>0}du\; dlogt \end{aligned}$$

that we may expect to see the process $Y(t)$ as a scaled homogeneous process evaluated in log time.

- Jeanblanc, Pitman and Yor (2001) show that this is indeed the case and we may write for example that

$$Y(t) = Y(1) + \int_{1}^{\log(t)} e^{\gamma s} dU(s)$$

for a Lévy process $U(t)$ that one constructs from the additive process $Y(t)$.

- We may regard $U(t)$ as the underlying uncertainty in the economy that has been time changed by the logarithm and scaled by the exponential.

- The process $U(t)$ is in fact an underlying $BDLP$ in the sense defined by Barndorff-Nielsen and Shepard. Specifically one may construct a stationary solution to the OU equation

$$dZ = -\gamma Z dt + dU$$

and relate $Y(t)$ to this stationary process as shown by

$$Y(t) = t^{\gamma} Z_{\log(t)}.$$

The Stock Price Models

- Our Stock price models are formulated in terms of our additive processes as discounted exponential martingales in the form
$$S(t) = S(0)\frac{\exp((r-q)t + Y(t))}{E\left[\exp\left(Y(t)\right)\right]}$$
where the normalizing expectation may be explicitly obtained from the characteristic function of the additive process.
- We investigate in this study 8 Sato processes from the perspective of their use for Structured Product Valuation.
- These are associated with unit time laws of $VG, NIG, Meixner, GH$ and $CGMY$ with four settings for Y.
- They all fit most underlier surfaces most of the time quite well and equally well, as documented in Carr, Geman, Madan and Yor (Mathematical Finance 2007).

Comparison of forward and spot Sato return distributions

- The Sato process fits the spot surface or the surface seen by the process at time zero very well.
- However, the process is inhomogeneous and creates internal forward return distributions that may go far afield from its own spot surfaces.

- Let $\phi_Y(u)$ be the characteristic function of the unit time self decomposable random variable used to generate the Sato process.
- The characteristic function of the log forward return

$$\ln\left(\frac{S(t+h)}{S(t)}\right)$$

is independent of the entire past and is given by

$$\exp\left(iu(r-q)h - \ln\phi_Y(-i(t+h)^\gamma) + \ln\phi_Y(-it^\gamma)\right)$$
$$\times\frac{\phi_Y(u(t+h)^\gamma)}{\phi_Y(ut^\gamma)}.$$

- This may be used to price forward starting options paying at time $t+h$ the value

$$100\left(\frac{S(t+h)}{S(t)} - a\right)^+$$

with the price

$$w_t(a,h)$$

for the gross return strike a and the maturity h.

Forward Return Assessments

- We ask three questions?
  - How well does a spot Sato process explain its own forward option prices?
  - What is the distance between the spot and forward Black Merton Scholes implied volatility curves?
  - What do these curves look like in comparison to the spot curve?
- We report the fit statistics of 8 Sato processes to the spot surface of SPX 20060517 and to the model internal one, two and five year forward surfaces.
- For the spot surface all the models fit the 230 options of the spot surface at a comparable level.

- We next construct the forward option price surfaces $w_t(a, h)$ for $t = 1, 2, 5$ years, and we construct three surfaces using 21 strikes a ranging from 80 to 120 in 2 dollar intervals, and four maturities h, ranging from $.25$ to one year in steps of $.25$. This gives us a total of 84 options for each of the three forward dates.
- We then fit each Sato process model at time 0, to each of its three forward surfaces. The fit statistics of all eight models on are presented in Tables 2, 3 and 4.
- We observe a deteriotiation in the APE as we move to the forward surfaces.
- From the graphs we see that the forward curves are closer to each other than they are to the spot curve.

TABLE 1
Spot Surface Fit Statistics

Model	RMSE	AAE	APE
VGSSD	0.6942	0.5502	0.0331
CGMYSSD1	0.6905	0.5468	0.0328
CGMYSSD2	0.6968	0.5525	0.0332
CGMYSSD3	0.7129	0.5662	0.0340
CGMYSSD4	0.7767	0.6073	0.0365
NIGSSD	0.7084	0.5636	0.0338
MXNRSSD	0.6889	0.5471	0.0328
GHSSD	0.6892	0.5466	0.0329

TABLE 2

One Year Forward Surface

Model	RMSE	AAE	APE
VGSSD	0.0907	0.0715	0.0537
CGMYSSD1	0.0991	0.0814	0.0395
CGMYSSD2	0.0859	0.0718	0.0348
CGMYSSD3	0.1459	0.1223	0.0594
CGMYSSD4	0.1426	0.0943	0.0457
NIGSSD	0.0693	0.0561	0.0272
MXNRSSD	0.0783	0.0638	0.0309
GHSSD	0.0626	0.0494	0.0240

TABLE 3

Two Year Forward Surface

Model	RMSE	AAE	APE
VGSSD	0.0898	0.0716	0.0596
CGMYSSD1	0.1051	0.0863	0.0447
CGMYSSD2	0.0869	0.0694	0.0359
CGMYSSD3	0.1683	0.1378	0.0709
CGMYSSD4	0.2056	0.1492	0.0764
NIGSSD	0.0758	0.0605	0.0312
MXNRSSD	0.0905	0.0752	0.0389
GHSSD	0.0848	0.0682	0.0353

TABLE 4

Five Year Forward Surface

Model	RMSE	AAE	APE
VGSSD	0.0783	0.0623	0.0640
CGMYSSD1	0.3362	0.2478	0.1453
CGMYSSD2	0.1319	0.1042	0.0611
CGMYSSD3	0.2046	0.1621	0.0929
CGMYSSD4	0.2882	0.2166	0.1217
NIGSSD	0.1018	0.0803	0.0461
MXNRSSD	0.1248	0.1034	0.0603
GHSSD	0.1290	0.1061	0.0622

- We next measure distances between spot and forward implied volatility curves.

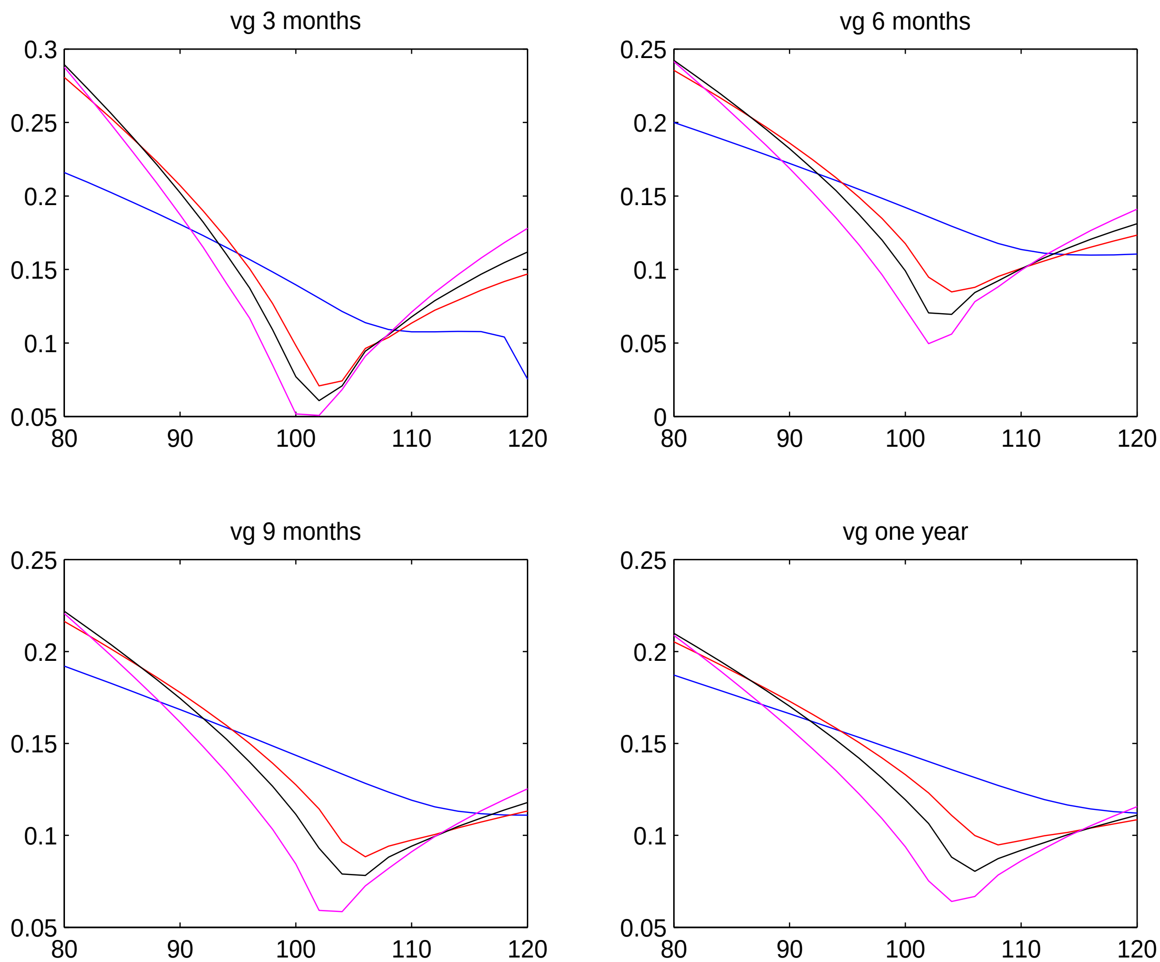
TABLE 5
RMSE of Spot and Forward Implied Volatility Curve

Model	One Year	Two Year	Five Year
VG	0.0255	0.0323	0.0407
CGMYp25	0.0534	0.0551	0.0555
CGMYp5	0.0524	0.0534	0.0537
CGMYp75	0.0508	0.0517	0.0517
CGMY1p25	0.0494	0.0504	0.0525
NIG	0.0511	0.0519	0.0517
GH	0.0536	0.0549	0.0549
MXNR	0.0534	0.0544	0.0538

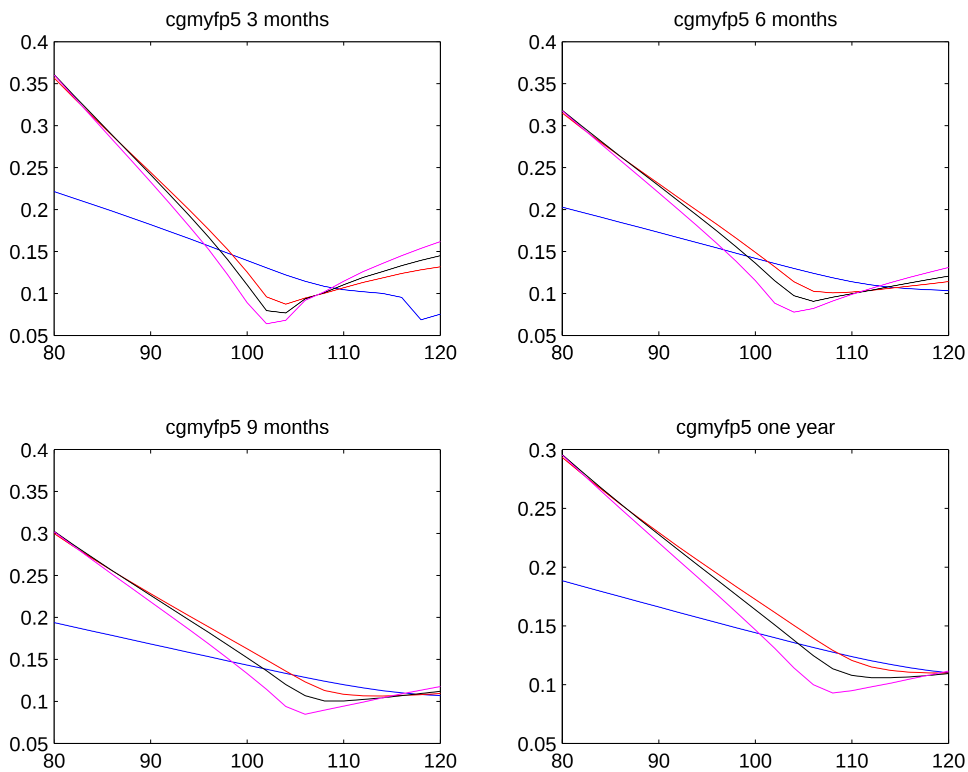
Realized Variance Options and Sato Processes

- Consider the continuous time realized quadratic variation in log prices to date t in the absence of a continuous martingale component.
- This is given by

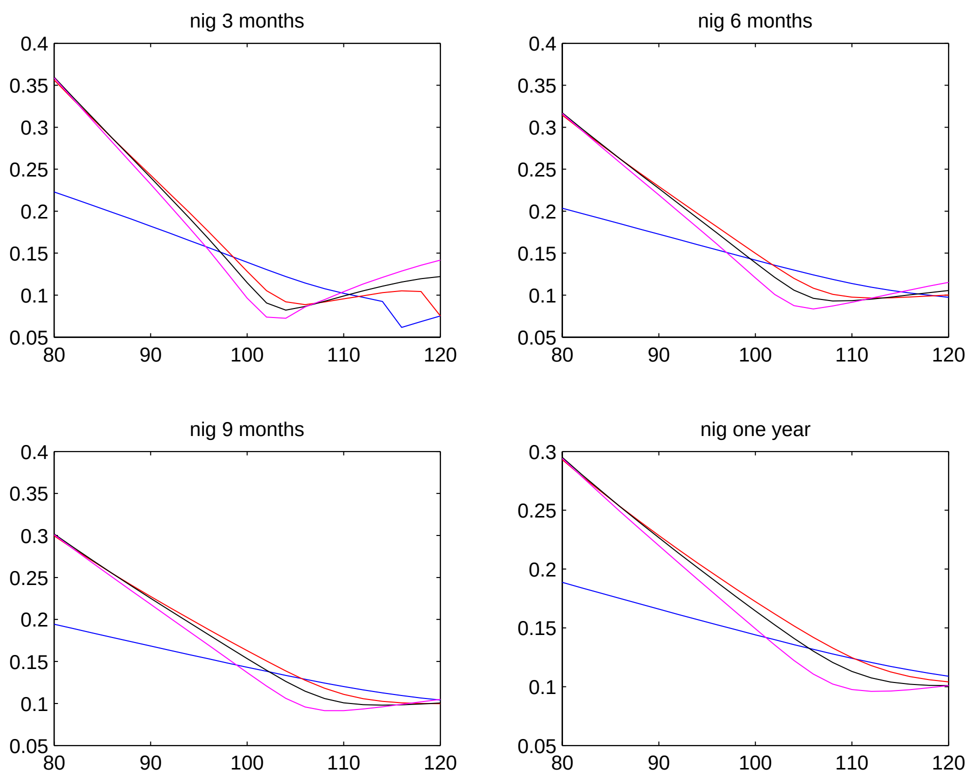
$$
\begin{aligned}
q(t) &= \frac{V(t)}{t} \\
V(t) &= \sum_{s \le t} (\Delta Y(s))^2
\end{aligned}
$$



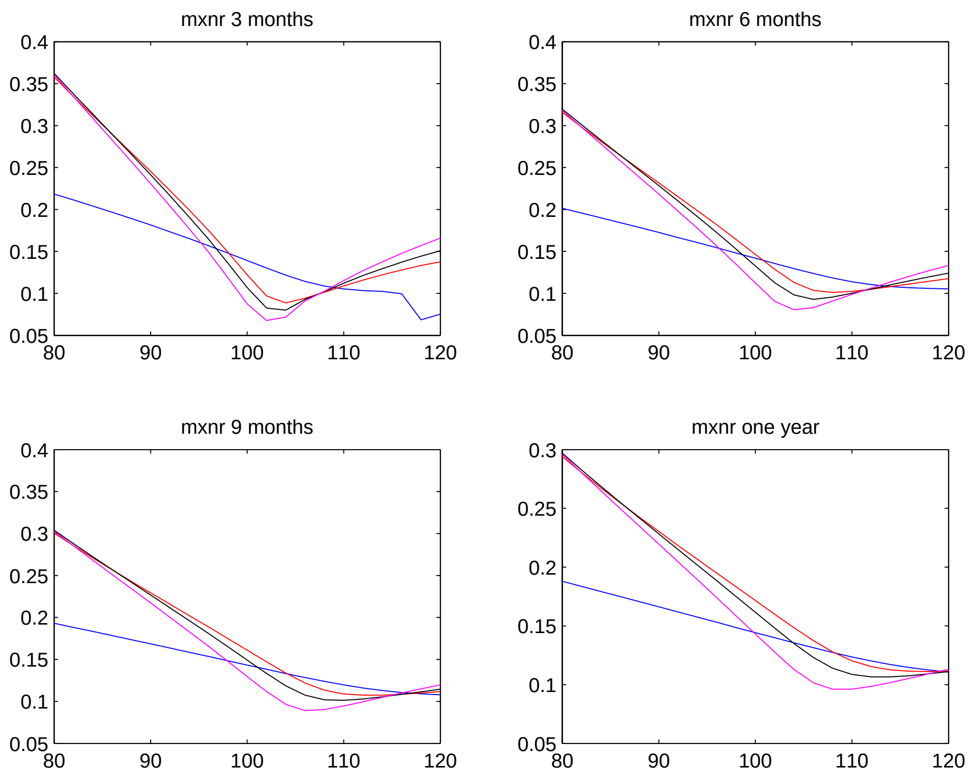
3.Forward Implied Volatilities for the VG Sato Process.



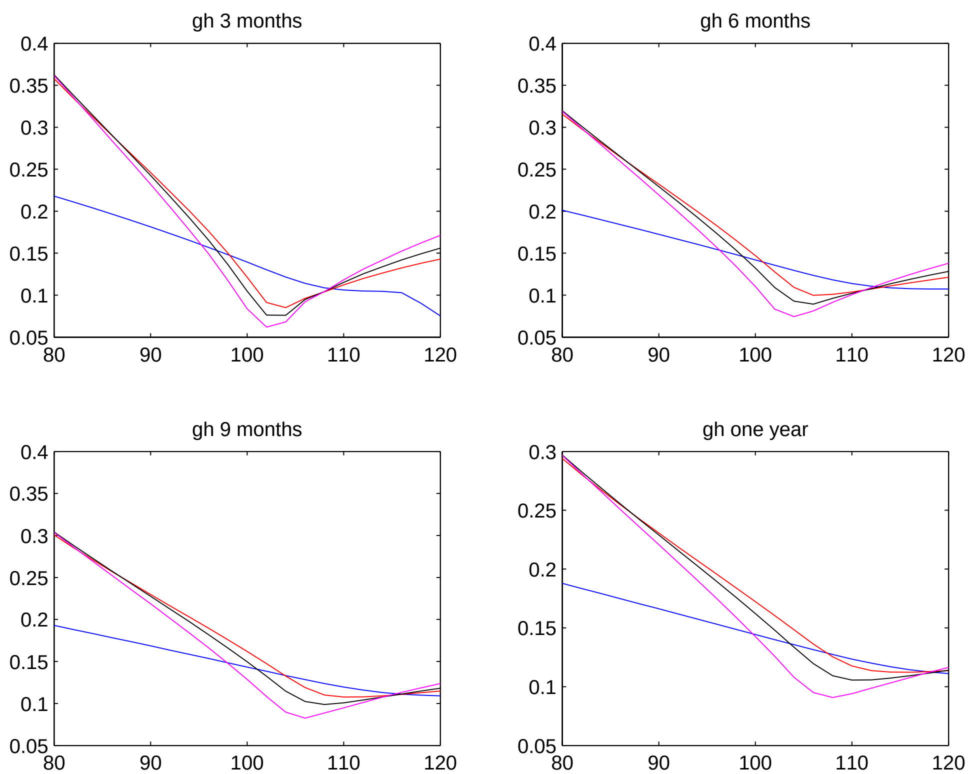
4.Forward Implied Volatilities for CGMY Sato process with Y=0.5



5.Forward Implied Volatilities for NIG Sato process



6.Forward Implied Volatilities for Meixner Sato process



7.Forward Implied Volatilities for GH Sato process

- $V(t)$ is itself a Lévy process when $Y(t)$ is a Lévy process with Lévy density $g(y)$, and the Lévy density of $V(t)$ is
$$v(y) = \frac{g(\sqrt{y})}{2\sqrt{y}} + \frac{g(-\sqrt{y})}{2\sqrt{y}},\ y > 0.$$
- $V(t)$ is itself an additive process when $Y(t)$ is an additive process with inhomogeneous Lévy density $g(y,t)$ and the inhomogeneous Lévy density of $V(t)$ is
$$v(y,t) = \frac{g(\sqrt{y},t)}{2\sqrt{y}} + \frac{g(-\sqrt{y},t)}{2\sqrt{y}},\ y > 0.$$

- In either case the variance of $q(t)$ may be inferred from the Laplace transform in the inhomogeneous case of $E[\exp(-\lambda q(t))] =$
$$\exp\left(-\int_0^t \int_0^\infty \left(1 - e^{-\lambda y/t}\right) v(y,u) dy du\right)$$
- For a Lévy process this variance is as expected decreasing with t and it is
$$\frac{\int_0^\infty y^2 v(y) dy}{t}$$
- For a Sato process it can be shown to be
$$\left(-\int_0^\infty w^4 h'(w) dw\right) \frac{t^{2(2\gamma-1)}}{2}$$
- and is constant for $\gamma = 1/2$.
- As a result long dated out of the money options on variance have no value under a Lévy process while they may have value for a Sato process.

Ziggurat Simulation of Sato processes

- We know that the Sato process can be written in the form

$$\ln(S(t)) = \ln(S(0)) + a(t) + \sum_{s \leq t} s^{\gamma} \Delta U(s)$$

where the process $U(t)$ is additive with an inhomogeneous Lévy density of the form

$$l(u,t) = \mathbf{1}_{u>0} \left(-\frac{\gamma h'(u)}{t} \right) + \mathbf{1}_{u<0} \left(\frac{\gamma h'(u)}{t} \right)$$

- Hence for some time dependent Poisson arrival rate we need to simulate the jump sizes from a decreasing density on the positive side to generate the absolute value of positive jumps u_p and negative jumps u_n with N_p, N_n being the random number of jumps on each side.
- We then have

$$U(s+h) - U(s) = \sum_{j=1}^{N_p} u_{pj} - \sum_{k=1}^{N_n} u_{nk}$$

- The Ziggurat method of Marsaglia and Tsang (1984) is ideally suited for such a simulation.
- The method is an acceptance rejection method where we first cover the density function by K rectangles of equal area and use a uniform draw to choose a rectangle, and two independent uniform draws to choose a point (x, y) in the rectangle and we accept x just if y is below the density $f(x)$.
- We only have to test for acceptance when x is above the right end point of the rectangle above the chosen rectangle.
- By increasing the number of rectangles we can make the rejection region small.
- We only need a form for the tail of the density in the lowest rectangle.

Models with calibration Details

- All calibrations are to SPX 20060517 with 230 option prices
- For HSV we have

$$
\begin{aligned}
dS &= (r-q)Sdt + \sqrt{v(t)}S(t)dW_S(t) \\
dv &= \kappa(\theta^2 - v)dt + \lambda\sqrt{v(t)}dW_v(t) \\
dW_S dW_v &= \rho dt \\
v(0) &= .1424 \\
\theta &= .1564 \\
\kappa &= 3.1140 \\
\lambda &= .4764 \\
\rho &= -.7124 \\
APE &= .0288
\end{aligned}
$$

- For SVJ we have

$$
\begin{aligned}
dS &= (r-q)S(t)dt + \sqrt{v(t)}S(t)dW_S(t) \\
+S(t\_) & \int_{-\infty}^{\infty} (e^x - 1)\left(\mu(dx, dt) - \lambda_J k(x) dx dt\right) \\
dv &= \kappa\,(\eta - v)\,dt + \lambda\sqrt{v(t)}dW_v(t) \\
dW_S dW_v &= \rho dt \\
k(x) &= \frac{1}{\sigma_J\sqrt{2\pi}}\exp\left(-\frac{(x-\mu_J)^2}{2\sigma_J^2}\right) \\
v(0) &= .02064 \\
\lambda_J &= 2.52 \\
\mu_J &= .000574 \\
\sigma_J &= .0155 \\
\kappa &= 2.701 \\
\lambda &= .5147 \\
\rho &= -.6987 \\
APE &= .0283
\end{aligned}
$$

- For VGSA we have

$$
\begin{aligned}
S(t) &= S(0)\exp((r-q)t)\frac{\exp(X(Y(t))}{E[\exp(X(Y(t))]} \\
Y(t) &= \int_0^t y(u)du \\
dy &= \kappa(\theta - y(t))dt + \lambda\sqrt{y(t)}dW_y(t)
\end{aligned}
$$

where $(X(t), t \geq 0)$ is the two parameter VG process with $Lévy$ measure

$$
\begin{aligned}
k(x) &= \exp(Ax - B|x|) \\
A &= (G-M)/2 \\
B &= (G+M)/2 \\
y(0) &= 10.8277 \\
G &= 24.9927 \\
M &= 46.0252 \\
\kappa &= 2.7896 \\
\theta &= 5.9009 \\
\lambda &= 7.1422 \\
APE &= .0345
\end{aligned}
$$

- Local Volatility and Local Lévy were generated from $VGSSD$ with

$$\begin{aligned} \sigma &= .1218 \\ \nu &= .6566 \\ \theta &= -.1209 \\ \gamma &= .5326 \\ APE &= .0331 \end{aligned}$$

- Local Lévy localized $CGMY$ with $G = 5, M = 10, Y = .5$.

- For Local Volatility we have

$$dS = (r - q)S(t)dt + \sigma(S(t), t)dW(t)$$

with a nonparametric local volatility surface $\sigma(S, t)$ and $(W(t), t \geq 0)$ a driving Brownian motion. We recover local volatility from

$$\sigma^2(K, T) = 2\frac{C_T + qC + (r - q)KC_K}{K^2 C_{KK}}$$

- For local Lévy we have

$$dS = (r-q)S(t)dt$$
$$+S(t\_) \int_{-\infty}^{\infty} (e^x - 1)\left(\mu(dx,dt) - a(S(t\_),t)k(x)dxdt\right)$$

- The specific Lévy measure we localize is $CGMY$ where the C parameter is absorbed by the speed function and

$$k(x) = \mathbf{1}_{x<0}\frac{\exp(-5|x|)}{|x|^{1.5}} + \mathbf{1}_{x>0}\frac{\exp(-10x)}{x^{1.5}}$$

  The nonparametric speed function may be recovered on solving the convolution equation

$$
\begin{aligned}
\int_{-\infty}^{\infty} b(y,t)\psi_e(k-y)dy &= c_T + qc + (r-q)c_k \\
c(k,T) &= C(e^k, T) \\
\psi_e(z) &= \mathbf{1}_{z<0} \int_{-\infty}^{z} \left(e^z - e^x\right) k(x)dx + \\
&\quad \mathbf{1}_{z>0} \int_{z}^{\infty} \left(e^x - e^z\right) k(x)dx \\
a(K,T) &= \frac{b(\ln(K), T)}{K^2 C_{KK}}.
\end{aligned}
$$

- The Sato processes employed were $CGMYSSD$ with $Y = .25, .5, .75$ and $MeixnerSSD$.

  For $CGMYSSD$ we have
  $$h(x) = \mathbf{1}_{x>0}\frac{Ce^{-Mx}}{x^Y} + \mathbf{1}_{x<0}\frac{Ce^{-G|x|}}{|x|^Y}$$

-

Y	.25	.5	.75
C	.7461	.3673	.1829
G	7.0953	5.9112	4.8074
M	25.46	28.3871	38.7755
γ	.5329	.5332	.5337
APE	.0328	.0332	.0340

- For $MeixnerSSD$ we have

$$\begin{aligned} h(x) &= d\frac{\exp\left(\frac{b}{a}x\right)}{\left|\sinh\left(\frac{\pi x}{a}\right)\right|} \\ a &= .1755 \\ b &= -1.7765 \\ d &= .6349 \\ \gamma &= .5329 \\ APE &= .0328 \end{aligned}$$

The Products Priced

- Monthly reset cliquets with annual maturities up to 5 years and daily monitored options on variance and volatility with annual maturities up to 5 years.
- The cliquets are arithmetic and locally floored and globally capped $LFGC$ or locally capped and globally floored $LCGF$.
- In addition we consider swing and reverse swing cliquets, SC, RSC.
- Let R_n denote the monthly return computed as a percentage change. The cliquet payoffs are by path ω as follows.

$$
\begin{aligned}
LFGC(\omega) &= Min\left[\sum_n (R_n \vee LF), GC\right] \\
LCGF(\omega) &= Max\left[\sum_n (R_n \wedge LC), GF\right] \\
SC(\omega) &= Min\left[\sum_n \left(|R_n| - k\right)^+, GC\right] \\
RSC(\omega) &= Min\left[\sum_n \left(k - |R_n|\right)^+, GC\right]
\end{aligned}
$$

- The payoffs for the variance and volatility options are

$$VarOpt(\omega) = 10000 \left(\frac{252}{T} \sum_{t=1}^{T} R_t^2 - k^2 \right)^+$$

$$VolOpt(\omega) = 100 \left(\sqrt{\frac{252}{T} \sum_{t=1}^{T} R_t^2} - k \right)^+$$

Remarks on Prices

LFGC

- The $Sato$ processes give a higher value for these cliquets with SVJ and LL giving values closer to these from among the other processes.
- Local volatility gives the lowest values excepting the large local floor of -15%.
- In each case the values rise with maturity and the global cap and fall as we lower the local floor.
- The sharper skews for forward returns in Sato processes probably account for the higher cliquet prices.

Locally Floored and Globally Capped Cliquets

LF		Global Cap 25				Sato processes			
	HSV	SVJ	VGSA	LV	LL	Y=.25	Y=.5	Y=.75	MXNR
	6.51	7.18	6.33	5.97	6.95	7.83	7.81	7.84	8.07
	11.17	11.91	10.19	10.28	11.87	13.24	13.33	13.35	13.39
-5	13.79	14.35	12.26	13.06	14.87	16.24	16.25	16.35	16.28
	15.04	15.56	13.32	14.61	16.37	17.62	17.52	17.61	17.48
	15.60	16.05	13.94	15.27	16.85	17.93	17.86	17.92	17.76
	3.49	4.06	3.83	3.14	4.77	5.16	5.21	5.13	5.29
	5.86	6.40	5.96	5.29	8.22	9.41	9.50	9.27	9.47
-10	7.18	7.44	6.91	6.86	10.43	12.17	12.14	11.96	12.17
	7.83	8.13	7.36	7.87	11.71	13.89	13.70	13.59	13.74
	8.30	8.63	7.73	8.28	12.31	14.82	14.60	14.52	14.59
	2.77	3.24	3.09	2.72	3.76	3.93	4.02	3.94	3.97
	4.49	4.77	4.70	4.51	6.44	7.33	7.46	7.23	7.32
-15	5.29	5.14	5.24	5.75	8.11	9.65	9.68	9.43	9.60
	5.55	5.40	5.39	6.53	9.05	11.24	11.12	10.96	11.10
	5.79	5.60	5.58	6.74	9.45	12.27	12.08	11.95	12.07
		Global Cap 50							
	6.66	7.43	6.42	6.13	7.26	7.95	7.92	7.92	8.20
	12.99	14.21	11.26	11.59	13.29	14.38	14.48	14.53	14.73
-5	18.43	19.80	15.44	16.63	18.45	19.44	19.59	20.00	20.00
	22.53	23.97	18.83	20.70	22.75	23.30	23.55	24.30	23.96
	25.29	26.70	21.37	23.59	25.86	26.09	26.40	27.31	26.66
	3.59	4.23	3.91	3.29	5.06	5.27	5.31	5.21	5.41
	7.10	7.92	6.85	6.29	9.36	10.37	10.43	10.21	10.55
-10	10.11	10.88	9.38	9.24	13.07	14.65	14.63	14.57	14.95
	12.38	13.32	11.39	11.79	16.21	18.07	18.06	18.21	18.45
	14.17	15.12	12.94	13.61	18.54	20.73	20.72	20.95	21.03
	2.87	3.40	3.17	2.87	4.05	4.04	4.11	4.01	4.09
	5.67	6.20	5.58	5.50	7.51	8.25	8.33	8.14	8.33
-15	8.03	8.33	7.63	8.05	10.52	11.92	11.94	11.81	12.12
	9.75	10.11	9.22	10.30	13.03	14.96	14.96	14.97	15.23
	11.14	11.42	10.45	11.85	14.90	17.41	17.36	17.40	17.59

LCGF

- For the local cap of 5% the $Sato$ processes give a substantially higher value, excepting $VGSA$.
- For higher caps the values are closer with SVJ and LV giving among the highest values for the 15% local cap, though the $cgmysato(Y = .75)$ is close for this cap.
- As we raise the local cap the optionality disappears and we have an in the money situation with the calibrated models in basic agreement.
- For an effective cap the Sato processes have higher prices possibly due to lower at the money volatilities.
- The values rise as we raise the local cap and the maturity and fall as the global floor is dropped.

Locally Capped and Globally Floored Cliquets

Global Floor -25

						Sato Processes			
LC	HSV	SVJ	VGSA	LV	LL	Y=.25	Y=.5	Y=.75	MXNR
	0.48	0.53	2.44	0.43	0.74	2.35	2.50	2.81	2.51
	1.33	1.01	4.90	1.43	2.03	4.63	4.83	5.44	4.85
5	1.94	1.44	6.90	2.16	3.28	6.42	6.73	7.70	6.84
	2.45	1.81	8.53	2.95	4.31	7.99	8.41	9.57	8.47
	3.03	2.22	10.01	3.50	5.24	9.40	9.82	11.02	9.81
	2.82	3.10	3.35	3.12	2.29	3.11	3.18	3.29	3.24
	5.63	5.68	6.25	6.19	4.69	5.96	6.06	6.34	6.13
10	7.84	7.80	8.64	8.62	6.84	8.19	8.42	8.99	8.56
	9.79	9.70	10.64	10.84	8.60	10.08	10.46	11.22	10.52
	11.62	11.36	12.45	12.62	10.13	11.72	12.15	12.97	12.16
	3.22	3.62	3.46	3.38	2.85	3.25	3.28	3.32	3.34
	6.43	6.73	6.43	6.69	5.68	6.31	6.32	6.44	6.40
15	9.01	9.29	8.86	9.36	8.17	8.75	8.85	9.19	9.01
	11.27	11.60	10.92	11.83	10.24	10.81	11.07	11.51	11.13
	13.39	13.63	12.77	13.86	12.02	12.60	12.89	13.36	12.92

Global Floor -50

	-0.31	-0.36	1.87	-0.35	0.17	1.92	2.06	2.37	2.05
	-0.38	-1.00	3.75	-0.50	0.76	3.68	3.96	4.55	3.90
5	-0.46	-1.55	5.41	-0.59	1.56	5.21	5.58	6.46	5.58
	-0.44	-1.72	6.82	-0.29	2.28	6.62	7.06	8.19	7.05
	-0.20	-1.76	8.17	-0.14	2.91	7.95	8.40	9.60	8.29
	2.21	2.44	2.82	2.54	1.76	2.70	2.76	2.87	2.80
	4.44	4.33	5.22	4.95	3.60	5.06	5.24	5.50	5.24
10	6.33	5.91	7.32	7.02	5.44	7.07	7.36	7.84	7.39
	8.04	7.56	9.14	9.06	7.01	8.83	9.22	9.93	9.23
	9.75	9.09	10.87	10.72	8.36	10.41	10.87	11.68	10.80
	2.65	3.02	2.94	2.83	2.32	2.84	2.86	2.89	2.89
	5.35	5.53	5.40	5.58	4.62	5.42	5.51	5.60	5.52
15	7.66	7.66	7.56	7.99	6.85	7.66	7.81	8.04	7.86
	9.73	9.77	9.44	10.30	8.76	9.60	9.86	10.23	9.87
	11.77	11.74	11.22	12.27	10.39	11.33	11.65	12.10	11.59

SC

- For the 5% strike the $Sato$ processes give intermediate values when compared with the values of the other models.
- At higher strikes the $Sato$ processes give a higher value, excepting LL that remains comparable with the $Sato$ processes.
- The $Sato$ processes reach the tails more easily than the models dominated by diffusion components.
- The values fall with the strike and rise with maturity.
- The drop with strike is quite marked for the parametric models $(HSV, SVJ, VGSA)$ and LV. It is less so for LL and the $Sato$ processes.

Swing Cliquet

Strike	HSV	SVJ	VGSA	LV	LL	Sato Processes Y=.25	Y=.5	Y=.75	MXNR
	6.56	7.06	4.47	6.28	6.53	5.66	5.47	5.21	5.80
	11.43	12.18	7.23	10.64	10.47	9.12	9.02	8.62	9.26
5	14.63	15.30	9.40	13.32	12.89	11.24	11.12	10.79	11.33
	16.38	16.91	11.12	14.77	14.37	12.41	12.39	12.16	12.53
	17.16	17.51	12.35	15.65	15.17	12.94	13.02	13.01	13.17
	1.46	1.82	1.15	0.82	3.12	2.49	2.46	2.26	2.55
	2.93	3.73	1.92	1.68	5.33	4.88	4.75	4.32	4.88
10	4.26	5.42	2.62	2.48	6.94	6.64	6.38	5.90	6.62
	5.38	6.71	3.25	3.09	8.27	7.88	7.54	7.08	7.86
	6.26	7.78	3.76	3.74	9.25	8.65	8.32	7.95	8.66
	0.32	0.48	0.29	0.11	1.68	1.19	1.25	1.12	1.21
	0.69	1.07	0.48	0.30	2.93	2.78	2.74	2.45	2.76
15	1.05	1.61	0.68	0.60	3.87	4.07	3.93	3.58	4.09
	1.36	2.08	0.85	0.81	4.77	5.12	4.85	4.49	5.14
	1.60	2.49	0.99	1.07	5.47	5.86	5.52	5.21	5.89

	7.06	7.84	4.62	6.57	7.13	6.04	5.87	5.57	6.18
	13.57	15.15	7.81	12.36	12.56	10.69	10.50	9.96	10.83
5	18.97	20.98	10.60	16.97	16.65	14.05	13.86	13.35	14.25
	23.16	25.13	13.05	20.20	19.88	16.41	16.25	15.86	16.69
	26.06	27.91	15.10	22.68	22.23	17.91	17.87	17.73	18.35
	1.49	1.90	1.15	0.83	3.37	2.61	2.58	2.37	2.66
	3.07	4.07	1.95	1.80	6.07	5.48	5.32	4.85	5.48
10	4.56	6.07	2.69	2.86	8.24	7.84	7.53	6.98	7.87
	5.87	7.76	3.35	3.70	10.23	9.73	9.29	8.71	9.78
	6.95	9.21	3.90	4.54	11.80	11.11	10.64	10.11	11.21
	0.33	0.49	0.29	0.11	1.81	1.24	1.29	1.16	1.25
	0.70	1.10	0.48	0.33	3.25	3.03	3.01	2.71	3.03
15	1.07	1.67	0.68	0.77	4.40	4.66	4.50	4.13	4.70
	1.40	2.19	0.85	1.10	5.55	6.10	5.77	5.38	6.16
	1.65	2.63	0.99	1.44	6.48	7.22	6.80	6.41	7.30

RSC

- The reverse swing cliquet with a lower global cap was more like a bond and we raised the cap to see the differences between the models.
- Focusing on the larger global cap of 1000, we see that the $Sato$ processes are close to LL with the parametric models ($HSV, SVJ, VGSA$) and LV giving substantially lower values.
- The values rise with the global cap, the local payout strike and the maturity.

Reverse Swing Cliquets

Global Cap 500

Local Cap	HSV	SVJ	VGSA	LV	LL	Sato Processes Y=.25	Y=.5	Y=.75	MXNR
	27.93	27.84	33.94	24.92	39.18	37.35	35.90	33.86	35.33
	53.45	53.17	67.44	47.57	75.88	78.76	75.44	70.78	74.15
5	76.39	75.76	97.69	67.79	109.06	118.27	113.52	106.21	111.08
	96.89	96.17	124.83	86.07	138.75	154.79	148.89	139.36	145.46
	115.41	114.40	149.07	102.45	165.49	188.23	181.34	169.94	177.00
	79.38	78.86	87.55	76.24	92.47	90.97	89.67	87.72	88.87
	151.07	149.72	170.10	145.08	177.61	181.93	178.68	174.09	177.19
10	215.52	213.17	244.55	206.62	254.47	266.46	261.65	254.29	259.08
	273.21	270.34	311.30	261.93	323.33	343.63	337.51	327.81	334.01
	325.12	321.44	369.74	311.19	377.57	389.27	388.96	386.75	388.43
	135.29	134.51	143.76	132.59	147.96	146.68	145.45	143.58	144.53
	257.27	255.29	277.22	252.15	283.30	288.02	284.90	280.49	283.28
15	366.93	363.59	397.47	359.12	405.13	417.31	413.19	406.23	410.59
	408.62	407.72	409.36	405.47	409.36	409.37	409.37	409.37	409.37
	389.40	389.39	389.40	388.56	389.40	389.40	389.40	389.40	389.40

Global Cap 1000

	27.93	27.84	33.94	24.92	39.18	37.35	35.90	33.86	35.33
	53.45	53.17	67.44	47.57	75.88	78.76	75.44	70.78	74.15
5	76.39	75.76	97.69	67.79	109.06	118.27	113.52	106.21	111.08
	96.89	96.17	124.83	86.07	138.75	154.79	148.89	139.36	145.46
	115.41	114.40	149.07	102.45	165.49	188.23	181.34	169.94	177.00
	79.38	78.86	87.55	76.24	92.47	90.97	89.67	87.72	88.87
	151.07	149.72	170.10	145.08	177.61	181.93	178.68	174.09	177.19
10	215.52	213.17	244.55	206.62	254.47	266.46	261.65	254.29	259.08
	273.21	270.34	311.30	261.93	323.33	343.63	337.51	327.81	334.01
	325.12	321.45	370.96	311.19	385.05	413.62	406.35	394.58	401.93
	135.29	134.51	143.76	132.59	147.96	146.68	145.45	143.58	144.53
	257.27	255.29	277.22	252.15	283.30	288.02	284.90	280.49	283.28
15	366.93	363.59	397.47	359.12	405.37	418.06	413.42	406.25	410.69
	465.15	461.03	505.29	455.09	514.75	536.19	530.23	520.72	526.58
	553.35	548.14	601.68	540.58	612.71	642.89	635.72	624.13	631.18

VarOpt and VolOpt

- Options on variance are priced above options on volatility as expected.
- The values drop with the strike quite substantially for the parametric models ($HSV, SVJ, VGSA$).
- This drop is less marked for LV but is still quite substantial at the lower maturities when compared with LL.
- For the longer maturities LV and LL are comparable. The values rise with maturity for LV and fall for LL.
- The $Sato$ processes maintain value with both strike and maturity with the values not falling that fast with strike or maturity.
- This is presumably a consequence of the effects of scaling as outlined in the theoretical analysis of the variance of realized quadratic variation for $Sato$ processes.

Options on Variance

	HSV	SVJ	VGSA	LV	LL	Sato Processes Y=.25	Y=.5	Y=.75	MXNR
	11.8791	12.7620	8.9487	11.5133	14.1940	13.1588	13.3587	12.9085	13.3285
	12.0206	13.0562	7.7240	11.7427	13.5320	13.5212	13.8933	13.5326	13.9245
10	12.1142	13.2807	7.1442	12.2678	13.0491	13.7560	13.8767	13.9932	14.3499
	12.1654	13.3500	6.7817	12.6359	12.9323	13.9556	14.1271	14.2751	14.6620
	12.1563	13.3747	6.4981	12.8979	12.7962	14.0972	14.0347	14.4664	14.5286
	8.1392	9.3108	6.0495	6.8958	12.2783	11.1961	11.4310	10.8681	11.2451
	7.7291	9.1799	4.4159	7.2976	11.2930	11.5657	11.9597	11.5111	11.8985
15	7.4704	9.0933	3.5502	8.0429	10.5452	11.7767	11.8967	12.0204	12.3530
	7.2324	8.9292	2.9470	8.5832	10.2718	11.9774	12.1616	12.3059	12.6815
	6.9869	8.7466	2.4973	8.8597	9.9986	12.1395	12.0530	12.4923	12.5132
	4.8132	6.1330	3.6499	3.3338	10.5534	9.3995	9.7202	9.0711	9.3441
	4.0476	5.5808	2.0494	3.9071	9.2655	9.7601	10.2006	9.7872	10.0478
20	3.4764	5.1572	1.3040	5.0538	8.3185	9.9415	10.0947	10.3148	10.5328
	2.9698	4.6919	0.8544	5.8544	7.8956	10.1316	10.3980	10.5869	10.8478
	2.5095	4.2866	0.5553	6.2488	7.4968	10.3077	10.2756	10.7604	10.6199
	2.5277	3.6707	1.9767	1.2372	9.0655	7.8797	8.2572	7.5724	7.6912
	1.7801	3.0391	0.8328	2.0268	7.5714	8.1646	8.6804	8.3398	8.4910
25	1.2416	2.4830	0.3096	3.6436	6.4692	8.3364	8.5130	8.8676	8.9477
	0.8855	2.0342	0.1855	4.6138	5.9115	8.5224	8.8855	9.1145	9.2502
	0.6122	1.6399	0.0687	5.0554	5.4720	8.6939	8.7138	9.2896	8.9534
	1.1798	1.9485	1.0322	0.3217	7.8162	6.6335	6.9531	6.3033	6.3069
	0.6276	1.4898	0.2446	1.1083	6.1761	6.7945	7.3828	7.1109	7.1961
30	0.3646	0.9440	0.0000	2.9246	4.9938	6.9532	7.1784	7.6353	7.6253
	0.2147	0.5885	0.0000	3.9018	4.3778	7.1192	7.5799	7.8702	7.9161
	0.0982	0.5040	0.0000	4.3325	3.9207	7.3230	7.3614	8.0550	7.5051
	0.4894	0.9056	0.5195	0.0000	6.7858	5.5801	5.8307	5.2559	5.2037
	0.2666	0.5917	0.0000	0.5220	5.0295	5.6411	6.2995	6.0708	6.1135
35	0.0000	0.1407	0.0000	2.3457	3.7904	5.7883	6.0046	6.6057	6.5177
	0.0000	0.1235	0.0000	3.3295	3.1796	5.9442	6.4886	6.8007	6.7560
	0.0000	0.1093	0.0000	3.7683	2.7163	6.2026	6.2109	7.0003	6.2191

Options on Volatility

Strike	HSV	SVJ	VGSA	LV	LL	Sato Processes Y=.25	Y=.5	Y=.75	MXNR
	4.8266	5.3324	2.7537	4.8385	4.9675	4.4794	4.5474	4.4019	4.7096
	5.1094	5.7390	2.2129	4.9472	4.9559	4.7065	4.8432	4.6752	4.9447
10	5.2857	6.0455	1.9711	5.1765	4.8958	4.8567	4.9039	4.8789	5.1692
	5.4012	6.1938	1.8251	5.3062	4.9502	4.9719	4.9881	5.0528	5.3471
	5.4520	6.2870	1.7101	5.4434	4.9571	5.0258	4.9794	5.1583	5.3590
	1.7579	2.2164	0.9655	1.3239	2.8967	2.5301	2.5949	2.4169	2.6201
	1.6378	2.2174	0.5465	1.4473	2.6859	2.7050	2.8023	2.6046	2.8101
15	1.5643	2.2242	0.3646	1.6319	2.4817	2.7961	2.8206	2.7795	2.9930
	1.4906	2.1829	0.2563	1.7545	2.4266	2.8812	2.8782	2.9121	3.1386
	1.4121	2.1237	0.1871	1.8189	2.3504	2.9324	2.8683	2.9822	3.1371
	0.4964	0.7834	0.2830	0.2486	1.7557	1.4575	1.5462	1.3776	1.4859
	0.3605	0.6626	0.0935	0.3274	1.4770	1.5900	1.6733	1.5396	1.6331
20	0.2713	0.5785	0.0391	0.4789	1.2614	1.6416	1.6727	1.6751	1.7862
	0.2007	0.4855	0.0169	0.5946	1.1717	1.6992	1.7256	1.7717	1.8892
	0.1450	0.4102	0.0073	0.6566	1.0766	1.7419	1.7187	1.8157	1.8692
	0.1144	0.2368	0.0689	0.0285	1.1002	0.8672	0.9562	0.8172	0.8535
	0.0580	0.1645	0.0128	0.0720	0.8359	0.9482	1.0291	0.9504	0.9847
25	0.0287	0.1125	0.0019	0.1990	0.6458	0.9831	1.0119	1.0521	1.0929
	0.0147	0.0765	0.0007	0.2991	0.5545	1.0254	1.0709	1.1207	1.1687
	0.0071	0.0498	0.0001	0.3500	0.4845	1.0537	1.0536	1.1537	1.1367
	0.0213	0.0575	0.0161	0.0017	0.7138	0.5356	0.5929	0.4943	0.4979
	0.0061	0.0341	0.0010	0.0186	0.4842	0.5724	0.6472	0.6026	0.6124
30	0.0021	0.0140	0.0000	0.1122	0.3352	0.5956	0.6282	0.6796	0.6910
	0.0007	0.0055	0.0000	0.1877	0.2645	0.6232	0.6772	0.7335	0.7492
	0.0002	0.0040	0.0000	0.2254	0.2163	0.6514	0.6552	0.7615	0.7001
	0.0032	0.0109	0.0036	0.0000	0.4809	0.3365	0.3708	0.3069	0.3012
	0.0010	0.0048	0.0000	0.0037	0.2851	0.3504	0.4178	0.3904	0.3893
35	0.0000	0.0003	0.0000	0.0650	0.1712	0.3656	0.3886	0.4526	0.4486
	0.0000	0.0002	0.0000	0.1236	0.1239	0.3855	0.4395	0.4907	0.4860
	0.0000	0.0002	0.0000	0.1547	0.0922	0.4166	0.4135	0.5158	0.4268

Conclusion and Continuation

- As the market for structured products grows with an increase in the number and type of risks being traded the demand for models capable of evaluating these risks continues to grow.
- We continue to develop models with new risk dimensions not present in earlier generation models.
- The activity of answering the what if questions of the structured desks in equity, FX, fixed income, credit and hybrid products is placing a huge demand on the ability of quants to respond with fast and sensible algorithms capable of pricing, hedging, and risk managing the products being traded.

- Hopefully the internal divisions of investment houses in collaboration with departments of financial engineering will find solutions adequate enough to sustain the growth.
- There is a demand to construct models in which forward return distributions do not depart too far from the historical spot return experience.
  - We try to meet this request as best we can.
    - Such is clearly met by a Lévy process, but this will not fit the surface.
  - The question arises as to whether the request is reasonable.
- This is a question worth contemplating at both a theoretical and operational level.