Evolutionary Finance: A tutorial

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joint work with

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...

Idea

• Application of evolutionary dynamics (mutation and selection) to the analysis of the long-run performance of investment strategies (portfolio rules).

• A stock market is understood as a heterogeneous population of frequently interacting investment strategies in competition for capital.

• The general aim of the work is to build a "Darwinian theory" of portfolio selection.

Evolutionary ideas have a long tradition in economics/finance
(Santa Fe Institute in 80s and 90s)
Our model uses concepts going back to Marshall, Samuelson, Alchian, Shapley, Milnor, Shubik,...

Motivation

Market selection of investment strategies in financial economics mostly uses the conventional general equilibrium setting (Radner equilibrium):

> agents maximize discounted sums of expected utilities based on their beliefs or forecasts. All agree on all future prices and events: perfect foresight!

Blume and Easley (JET 1992, Ec. 2006, ...), Sandroni (Ec. 2000)

Drawbacks

• unobservable agents' characteristics (individual utilities or subjective beliefs)

• in simulations one needs to solve all agents' optimization problems simultaneously

• consumption and asset allocation decision intimately linked

Our view

In contrast we only assume

short-run equilibrium
(today's endogenous prices determined by market clearing)

and consider models based on **random dynamical systems**

to obtain investment strategies with certain **asymptotic properties**

Strengths

- only 'observables' are permitted
- selection criterion is survival ('unbeatable' in any pool)
- optimization (if any) based on observations (no future prices!)
- suitable for simulation studies
- gives theory closer to financial practice

Survey of the field: Evstigneev, Hens and Schenk-Hoppé (2009) *Evolutionary Finance*, Chapter 9 in *Handbook of Financial Markets: Dynamics and Evolution*, North-Holland

Results/Applications of our model

Identification of investment strategies ensuring

survival or, stronger, dominance (single survivor)

Computational laboratory to test performance of investment styles interacting in market (rather than testing on *given* return/price)

Q What happens if we let

Hersh Shefrin's behavioral guys compete in a market

or Stan Uryasev's CVaR optimizers, or all together?

Co-evolution of markets and behavior limit-order book market models with genetic programming and tournament selection.
Ladley, Lensberg, Schenk-Hoppé (2010) (if time permits)

Asset Market Games of Survival (Amir, Evstigneev, Schenk-Hoppé)

Game-theoretic model of asset market
(large investors with price impact)

Wealth dynamics
(survival and extinction – market selection)

Investment strategy = choice of portfolio weights

Investment funds **disclose** their portfolio allocation (with a lag)

e.g. EFA-sponsor **Skagen Funds**: SKAGEN Vekst

Every position, amount, purchase price, weight

Security	Investment (#)	Current value	Portfolio weight
Aareal Bank	NOK76m (500K)	NOK43m	0.51%
Hannover Re	NOK87m (435K)	NOK108m	1.27%
...	...	...	...
			$\sum = 100\%$

July 31, 2009 (published Aug. 10) [www.skagenfondene.no/category1978.html]

Model

Randomness $s_t \in S$ state of the world at date $t = 0, 1, 2, ...$ (exogenous process), history $s^t := (s_0, s_1, ..., s_t)$.

Assets $K \geq 2$ assets with payoffs $A_{t+1,k}(s^{t+1}) \geq 0$ and price $p_{t,k}$ (per unit). Supply of 1 unit each.

Investors $i = 1, ..., I$ hold portfolios $\theta_t^i \in R_+^K$.

Wealth dynamics

$$w_{t+1}^i = \sum_{k=1}^{K} \left[p_{t+1,k} + A_{t+1,k}(s^{t+1}) \right] \theta_{t,k}^i$$

Two things remain to be explained (and here comes the novelty):
- prices $p_{t,k}$ (short-term market clearing)

and
- portfolios $\theta_{t,k}^i$ (portfolio weights)

Prices and portfolios

Portfolio weights Each investor i chooses %-weights $(\lambda^i_{t,1}, ..., \lambda^i_{t,K})$ (≥ 0 and add up to 1). These are the investors' actions (decisions)

Given budget $b^i_t = \rho w^i_t$, the portfolio is $\theta^i_{t,k} = \dfrac{\lambda^i_{t,k} \, b^i_t}{p_{t,k}}$

Equilibrium Asset prices are given by 'supply = demand':

$$p_{t,k} = \sum_{i=1}^{I} \lambda^i_{t,k} b^i_t$$

Interpretation

Weights express an investor's opinion about 'correct' asset prices.

Market portfolio weight $\dfrac{p_{t,k}}{\sum_m p_{t,m}} < \lambda^i_{t,k} \Rightarrow$ investor i **overweight** in k.

Skagen Funds: "finding high quality companies at a low price" (undervalued)

Now comes game theory: Decisions can depend on **entire history**, i.e. *all* prices, portfolios and portfolio weights of all investors.

A strategy of investor i is a map Λ_t^i,

$$(\lambda_{t,1}^i, ..., \lambda_{t,K}^i) = \Lambda_t^i(s^t, p^{t-1}, \theta^{t-1}, \lambda^{t-1}) \quad \text{(portfolio weights).}$$

$\longrightarrow$ simultaneous-move I-person dynamic game

Market Dynamics One obtains a dynamics of prices, portfolios and wealth — for a given strategy profile Λ.

$$w_{t+1}^i = \sum_{k=1}^K [\rho \langle \lambda_{t,k}, w_{t+1} \rangle + A_{t+1,k}(s^{t+1})] \frac{\lambda_{t,k}^i w_t^i}{\langle \lambda_{t,k}, w_t \rangle}$$

Survival An investment strategy Λ^1 is a survival strategy, if

$$\inf_{t \geq 0} \frac{w_t^1}{\sum_j w_t^j} > 0 \quad (a.s.)$$

for any strategy profile $(\Lambda^2, ..., \Lambda^I)$. [unbeatable, no overtaking,...]

Examples of investment strategies

Constant "1/n" $\Lambda_t^1(...) = (1/K, ..., 1/K)$

Mimicking $\Lambda_t^1(...) = \lambda_{t-1}^2$

Performance-driven $\Lambda_t^1(...) = \lambda_{t-1}^i$ with i s.t. w_{t-1}^i / w_{t-2}^i maximal

Optimization of some objective $\lambda_t^i = \arg\max_\lambda f(s^t, p^{t-1}, \lambda)$

Examples of assets

Horse race (parimutuel betting market)

$$A_t(s^t) = \begin{bmatrix} \alpha_{t,1}(s^t) & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \alpha_{t,K}(s^t) \end{bmatrix}, \ \rho = 0$$

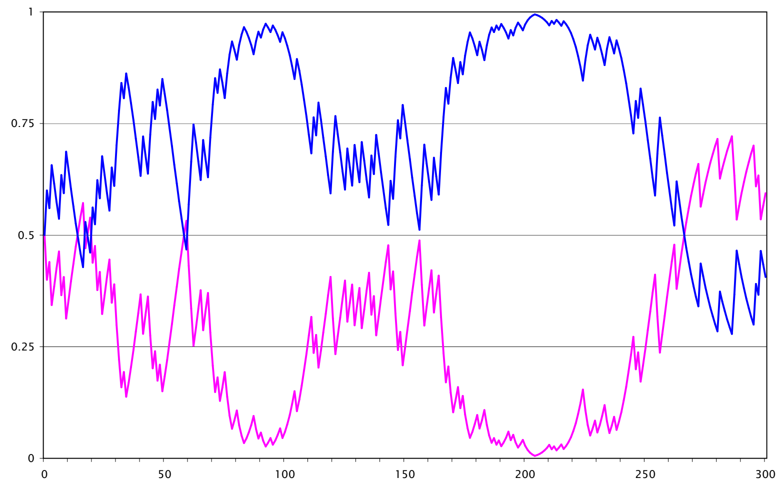
Incomplete asset market

$$A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 0 & 1 \end{bmatrix}, \ \rho \geq 0$$

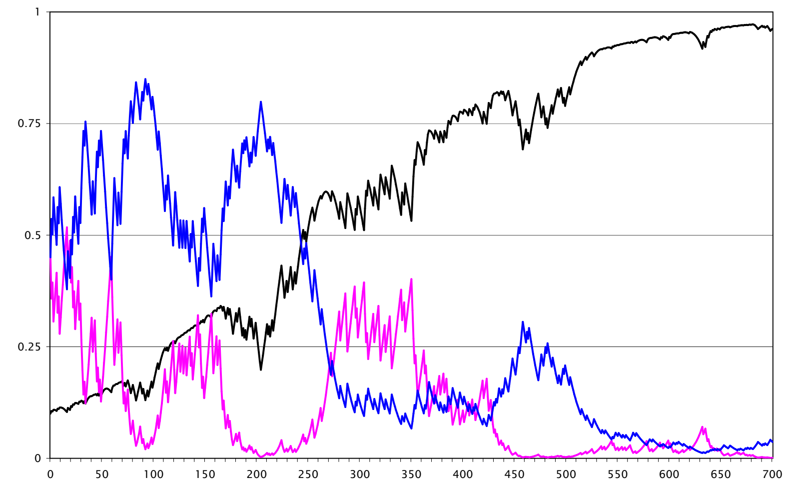
Simulation: Co-existence / extinction

$K = 2$, $S = 3$, i.i.d., uniform, $\rho = 0$, payoffs $A = \begin{pmatrix} 1 & 1 \\ 2 & 2 \\ 0 & 3 \end{pmatrix}$

Constant strategies
$\lambda^1 = (1/2, 1/2)$, $\lambda^2 = (1/4, 3/4)$, and $\lambda^* = (1/3, 2/3)$



2 strategies (λ^1 and λ^2)



3 strategies (λ^1, λ^2 and λ^*)

Can be fully understood by analyzing local stability
(co-existence: "your" prices turn against you)
Evstigneev, Hens, Schenk-Hoppé (2006) Economic Theory

Main Result

survival: $\inf_{t\geq 0} \frac{w_t^1}{\sum_j w_t^j} > 0\,(a.s.)$

The investment strategy Λ^* given by

$$\lambda_t^*(s^t) := (1-\rho)E\left(\sum_{l=1}^{\infty} \rho^{l-1}\widehat{A}_{t+l}(s^{t+l}) \mid s^t\right) \qquad [> 0]$$

with relative asset payoffs

$$\widehat{A}_{t+1,k}(s^{t+1}) := \frac{A_{t+1,k}(s^{t+1})}{\sum_{m=1}^{K} A_{t+1,m}(s^{t+1})}$$

is a survival strategy.

Assumption $E \ln \lambda_{t,k}^* > -\infty$

This strategy

- ensures a positive, bounded away from zero share of wealth (a.s.) **no matter what the other investors do**
- **basic**: only uses information on the history of the state
- **log-optimal** at its "own" prices
but not log-opt. in general (this needs returns = 'inf. from future')!

Relation to Kelly rule

The investment strategy Λ^* generalizes the well-known Kelly rule.

- betting your beliefs, expected payoffs, *no* returns enter!

Horse race ($\rho = 0$, $S = K$), 'iid' horses, objective probability of horse i wins $= \pi_i$, then

$$\lambda_t^*(s^t) = (\pi_1, ..., \pi_K)$$

this is the **classical Kelly rule** (1956).

Evstigneev, Hens and Schenk-Hoppé (2010)
"Survival and Evolutionary Stability of the Kelly Rule"
in *The Kelly Capital Growth Investment Criterion: Theory and Practice*
(editors: MacLean, Thorp and Ziemba)

Ed Thorp's bedtime reading:
Fortune's formula: The Untold Story of the Scientific Betting System That Beat the Casinos and Wall Street by William Poundstone

Uniqueness Result All basic survival strategies are "=" λ_t^*, i.e.

$$\sum_{t=0}^{\infty} \|\lambda_t^* - \lambda_t\|^2 < \infty \quad (a.s.)$$

Single survivor $\lim_{t\to\infty} \frac{w_t^1}{\sum_j w_t^j} = 1\,(a.s.)$

Stronger results when reducing the space of feasible strategies:

• constant, betting markets:

Evstigneev, Hens and Schenk-Hoppé (2002) Mathematical Finance

• basic, betting markets:

Amir, E, H and S-H (2005) Journal of Mathematical Economics

• constant, stock markets:

E, H and S-H (2009) Journal of Economic Theory

Q (open) Are there (proper) non-basic survival strategies?

Tricky!

For instance mimicking strategy does not work:

Betting market, ($\rho = 0$), $I = 2$, investor 1 mimics,
asset payoffs $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, iid, uniform.

Case 1
Investor 2 plays $\lambda^* = (1/2, 1/2)$. **Both survive**: $w_t^i \equiv w_1^i > 0$.

Case 2
Investor 2 plays $\lambda_t^2 = \begin{cases} (1/3, 2/3) & \text{for } t \text{ even} \\ (2/3, 1/3) & \text{for } t \text{ odd} \end{cases}$

No survivors: $\inf_t w_t^1 = \inf_t w_t^2 = 0$.

Extensions

- Supply $V_{t,k}(s^t) > 0$ rather than 1 unit, savings rate $\rho_t(s^t)$
- money market account (given interest)
- several currencies

Related research

- insurance companies (liquidity shocks): De Giorgi (2008) JEDC
- time step length $\to 0$: Palczewski and S-H (2010) JEDC

(mathematical finance with endogenous prices)

- market selection in cts model: P and S-H (2010) J. Math. Ec.

Mutation of strategies (add-on)

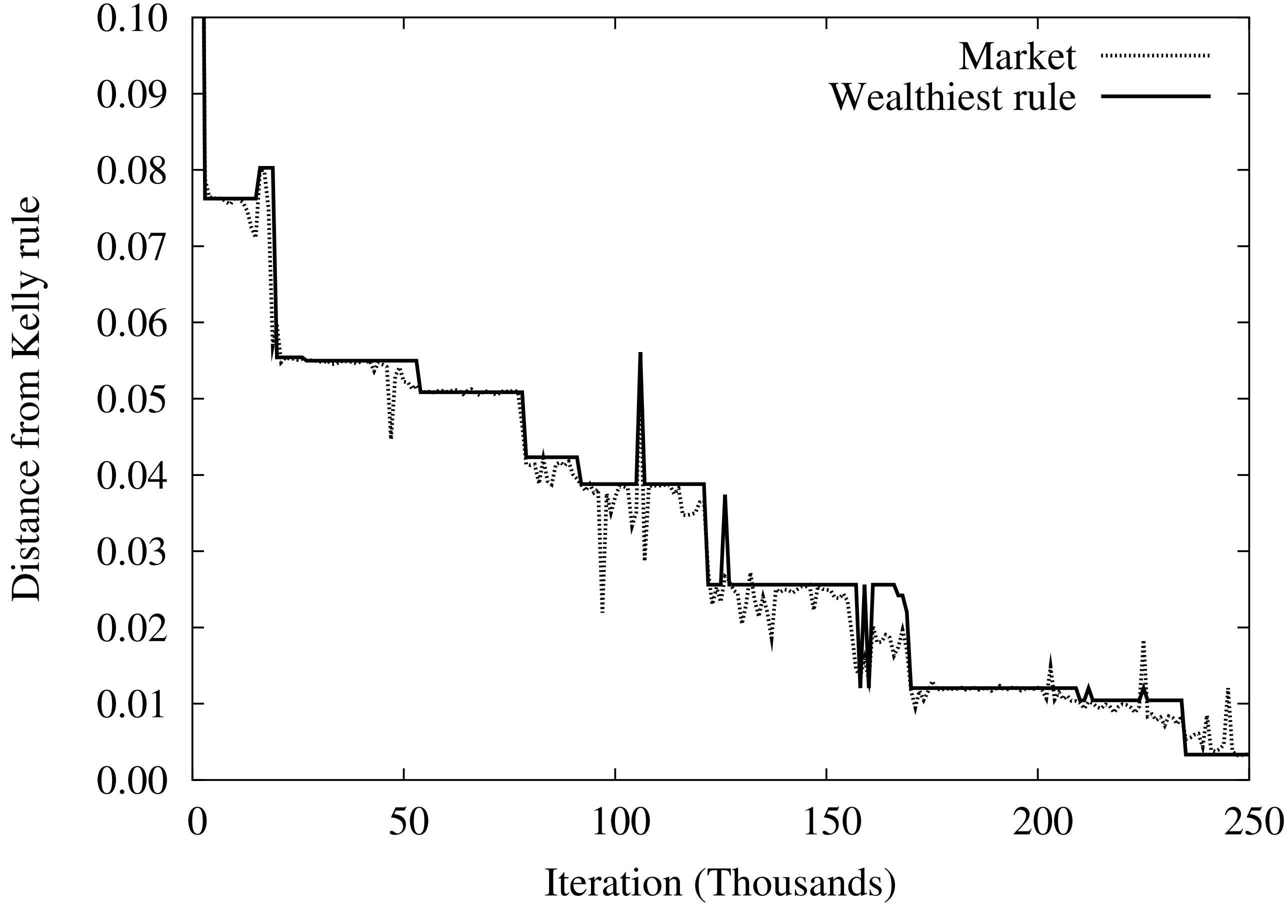
Strategies evolve through **mutation** rather than are a priori given

Ladley, Lensberg, Schenk-Hoppé (2008-2010 Norwegian grant)

Genetic programming with tournament selection

(This is number-crunching, we compete for computer time with climatologists)

Lensberg and Schenk-Hoppé (2007) Review of Finance



Distance: market prices resp. wealthiest strategy to Kelly rule

Computational Laboratory: Limit order book market

Ladley, Lensberg and Schenk-Hoppé (in preparation)

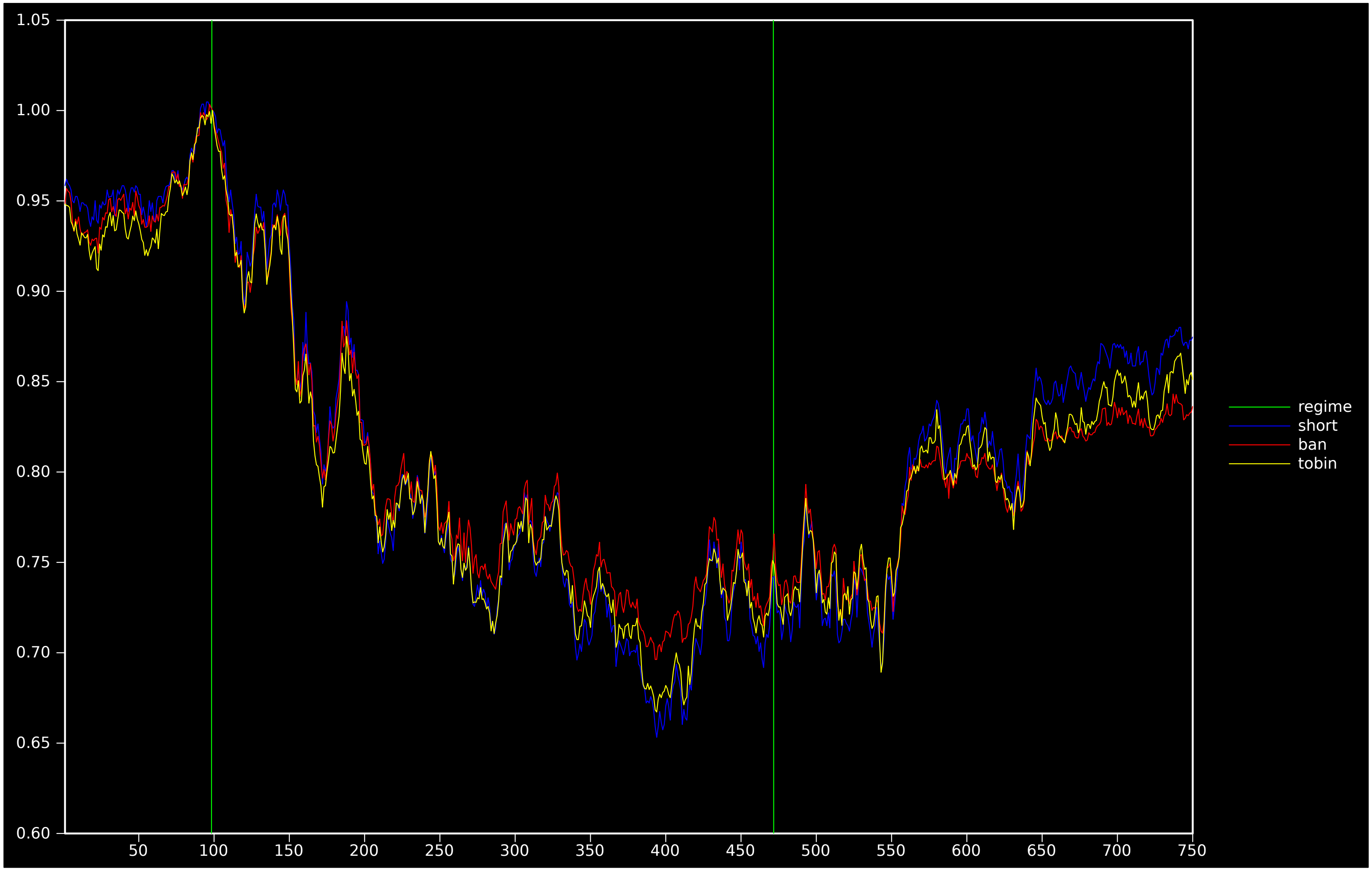
- Market structure: order book (with clearing house)
- Investment strategies: limit / market orders
- Investors: genetic programmes
- Tournament selection: wealth
- Regulation: margin requirements

Simulate to determine long-run outcome (market dynamics and investment behavior)

Insights into the co-evolution of investors and markets: Effects of regulation, time-series properties, resilience, ...

→ goes far beyond anything theory can currently handle

Simulation of artificial limit-order book market



Peak-trough analysis. Different regulatory frameworks (FED margin requirements (blue), Tobin tax (yellow), short-sale ban (red))

Conclusion

- Evolutionary finance offers a new way to model markets and analyze performance of investment strategies.
- In contrast to general equilibrium: observable variables only, no utility, no rational expectations...
- We revive the concept of survival games (Milnor/Shapley 50s) and find a survival strategy in financial markets.
- Ideas are fruitful in simulation studies of order-book markets

Contrary to conventional wisdom:

By designing a model close(r) to reality, life becomes simpler!