The Behavioralist and the Arb

Phil Maymin
NYU-Poly
CARISMA 2010

Behavioral Representative Investor

1. Evaluates based on cumulative prospect theory
2. Incorporates intermediate gains and losses

- Denote by $g(T)$ is the PDF of a T-period gamble
  - T independent coin tosses each paying $\begin{cases} u & \text{with prob. } \pi \\ d & \text{with prob. } 1-\pi \end{cases}$
- v[.] is the cumulative prospect theory value
- $A_i \in \{u,d\}$ is the realization of the asset at time i

$$v\left[g(T-t)+\sum_{i=1}^{t} A_i\right] = v\left[C_t\right]$$

- Now we know the certainty equivalent C_t

Prices

- Let's just look at scaled numbers for prices, rather than the raw certainty equivalent
- Subtract off the accumulated gains/losses and express relative to the remaining expected value

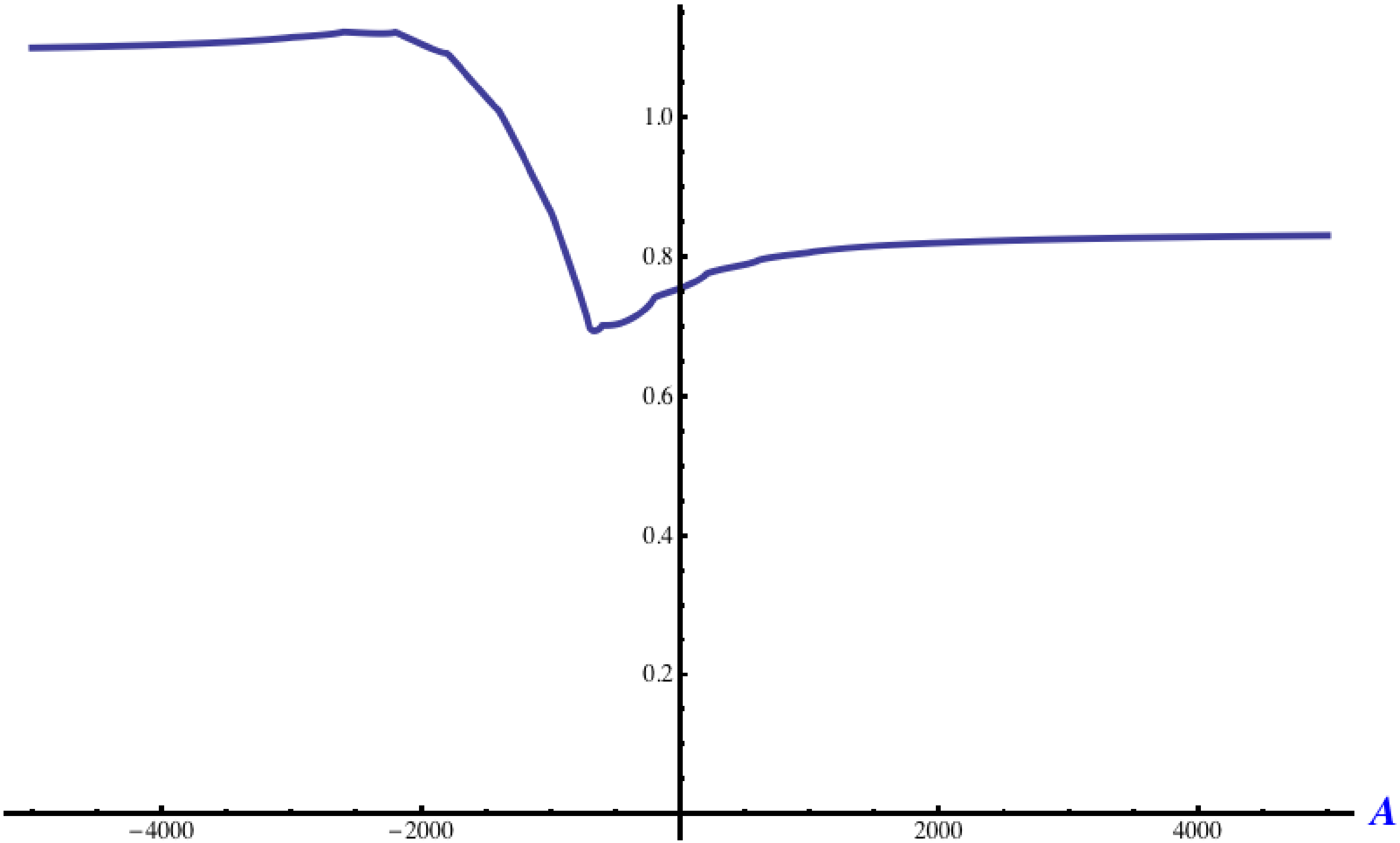
$$p_t = \frac{C_t - \sum_{i=1}^{t} A_i}{E\left[g(T-t)\right]} = \frac{C_t - \sum_{i=1}^{t} A_i}{(T-t)\left(\pi u + (1-\pi)d\right)}$$

A Numerical Example

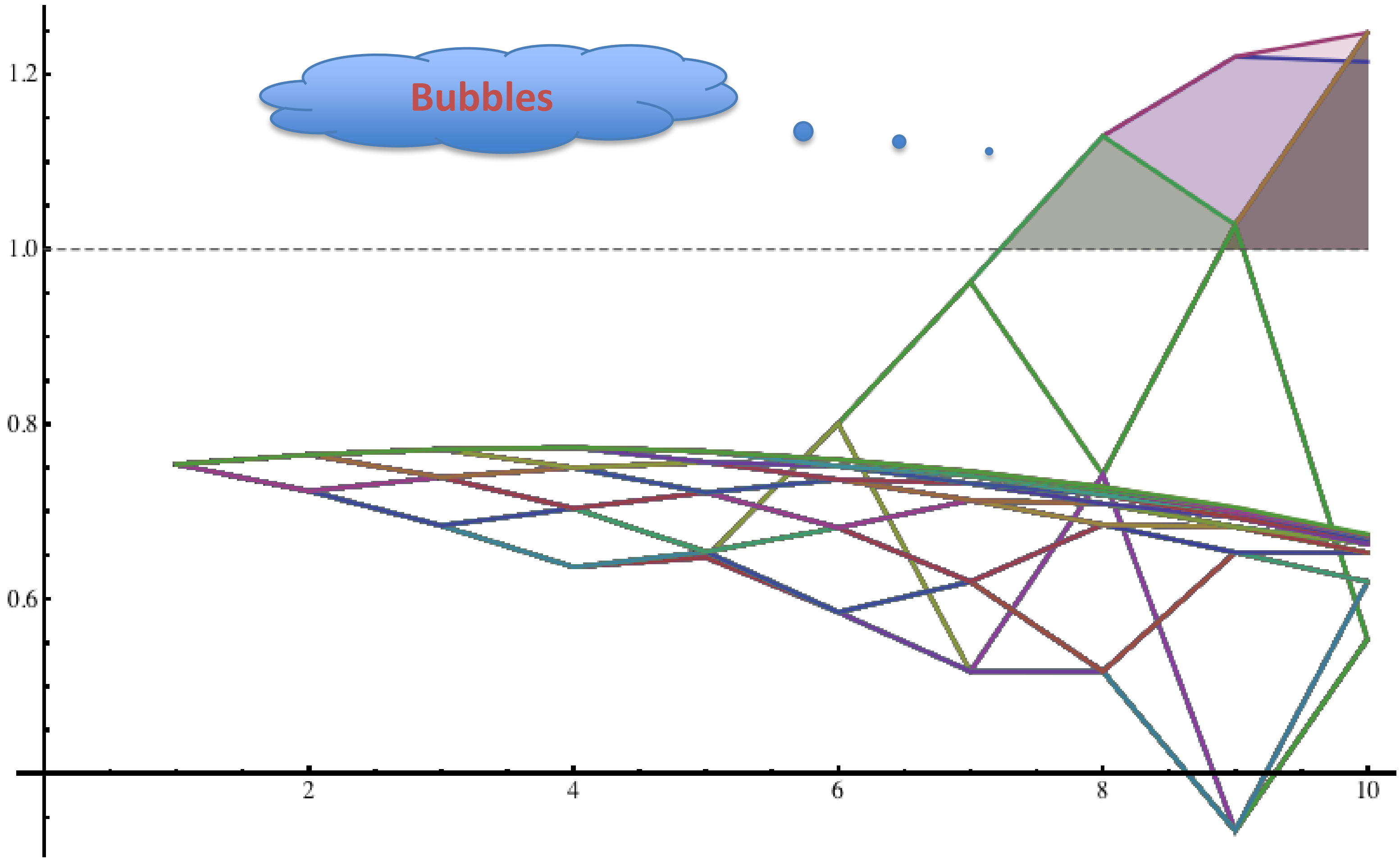
- $T = 10$
- $\pi = 0.5$, $u = 300$, $d = -100$
  - => $E[g(1)] = 100$
- Then:
  - $p_0 = 0.75$

  and
  - $p_1 = \begin{cases} 0.76 & \text{if } A_1 = u, \text{ with probability } \pi \\ 0.72 & \text{if } A_1 = d, \text{ with probability } 1 - \pi \end{cases}$
- Does the price always increase when the coin toss is up?

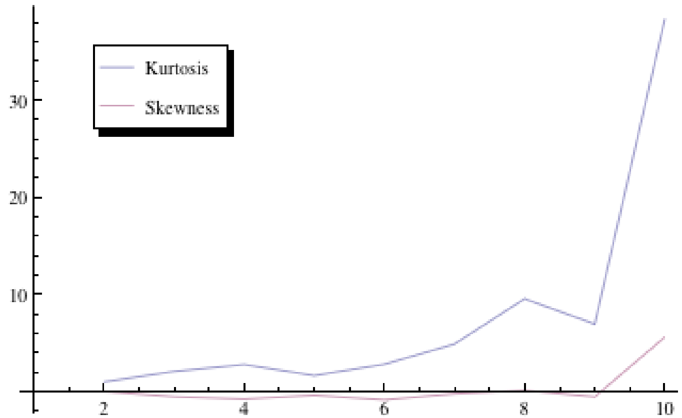
$v[g(10) + A] - v[A]$

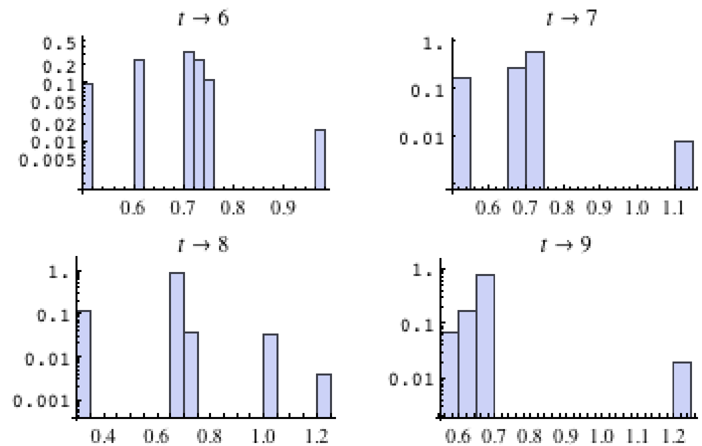


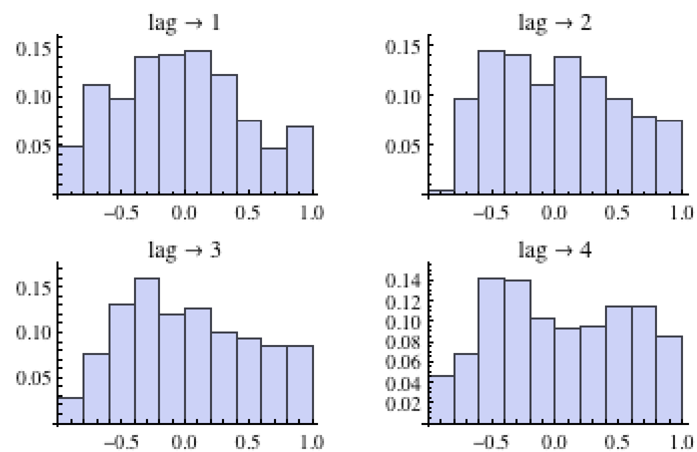
All Possible Price Paths for T = 10



Kurtosis, Skewness, and Autocorrelation







So far:

- Behavioral representative investor can generate realistic market price paths:
  - High kurtosis
  - High skewness
  - Momentum
  - Mean reversion
  - Bubbles (overpricing)

- What if behavioral representative investor sets prices, but price-taking arbitrageurs exist?
- Is arbitrage possible?

Arbitrage in this context

- Does there exist a *strategy* that takes as input the past (sliding) *n* price changes and gives as an output a trade signal, BUY or SELL, such that implementing that strategy across all paths is guaranteed to make a profit?

- E.g. if *n* = 3, there are eight distinct up/down histories, each of which can generate either a buy or a sell, so there are 2^(3+1) = 16 strategies.
- Does at least one of those strategies generate arb?

Results

- For same numerical example but
  - *T* ranging from 1 through 15
  - (implies #paths ranging from 1 to 16,384)
- Given *T*, try all possible *n* < T/3
- **RESULT:** <u>There is not a single arbitrage strategy</u>

- Does this always hold?

Alternative Numerical Examples

- Is it because it is binomial?
  - Simulate with normal distribution, same results.
- Is it because of the particular parameters?
  - Try u = 400 instead, same results.
- Is it because of the amount of time?
  - Try for even longer T's, out to T = 20, several million points to calculate: same results.

Conclusion

- Behavioral representative investor can generate realistic market price processes with high kurtosis, skewness, momentum, mean reversion, and bubbles, even if the underlying fundamental risk is independent and simple, like binomial or normal.
- It appears as if riskless profit is not possible by price-taking arbitrageurs.