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ASSET LIABILITY MANAGEMENT USING STOCHASTIC PROGRAMMING

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Asset-Liability Management (ALM) using Stochastic Programming (SP)

By Mehndi Pirbhai.

Mehndi Pirbhai was a research student in the Department of Mathematical Sciences, Brunel University. He worked with Professor Gautam Mitra in the field of financial models and risk management.

“A banker is a fellow who lends you his umbrella when the sun is shining and wants it back the minute it begins to rain”. (Mark Twain)

Scope

This white paper sets out to explain an important financial planning model called the *asset liability management* (ALM); in particular it discusses why in practice, optimum planning models are used. The ability to build an integrated approach which combines liability models with that of asset allocation decisions have proved desirable and more efficient in that it can lead to better ALM decisions. The role of uncertainty, and quantification of risk in these planning models is considered. This white paper will be of interest to *corporate treasurers*, to *fund managers* in the pension & insurance industry, and to *analysts* who support ALM models in different financial institutions.

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ALM: An Introduction

Many financial systems in the corporate as well as individual context are underpinned by a cash flow balancing (also called matching) activity. At an individual level typically a young professional may set up savings after child birth as he or she goes through the schools systems. The savings are assets suitably invested in bonds and shares and future payment for school fees are liabilities. At a corporate level many institutions take contributions from working employees of a corporation and invest these contributions by acquiring assets. These assets are, however, pledged to meet the pension payments of the individuals at future dates of their retirement. These pension payments are again the liabilities for the financial institution. A basic aspect of financial planning encompasses such matching activities of cash flows and is given the generic label: Asset and Liability Management or in short ALM. From a mathematical perspective these models can be set up in an equational form involving non-negative variables which represent inflow and outflow of funds and carry over of retained assets and funds from one planning period to the next.

So what more can be expected from ALM than the established techniques? To answer this, it is necessary to ascertain the pitfalls and difficulties encountered when making investment decisions. It is important to understand the risks that are borne when investing in a particular security or portfolio of securities. Generally the higher the risks undertaken, the higher the possible returns on that investment. But there are other constraints that cannot be ignored such as the nature of uncertainty in the decision process, taxes and transactions costs. There may also be legal guidelines and other policy requirements such as institution-specific rules on asset mix.

Returning to the fundamental aspect that any company has both assets and liabilities, it is clear that in the course of business the company will benefit from cash inflows and also have to meet liabilities. When asset streams are greater than liability streams there is a surplus and vice-versa when liability streams are greater than asset streams, there is a deficit (Figure 1 A company will always try to make sure that there is always a surplus but, in situations where there is a deficit, corrective measures can be taken to protect the company financially in the short-term. In the long term however, a company continuing to accumulate shortfalls is likely to be in a serious financial position and may be on the verge of insolvency). To avoid this financial quagmire, requires advanced and meticulous financial planning, and for large organisations ALM is invaluable. Indeed at time of going to press, Bethlehem Steel, the third-largest US steel maker has filed for Chapter 11 bankruptcy protection, its burgeoning pension liabilities being cited as one of the major reasons for the decision.

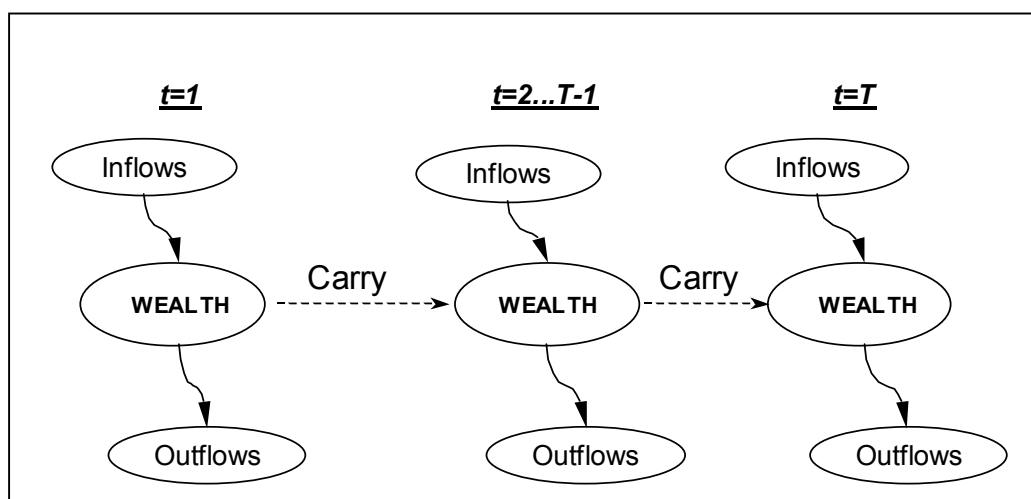


Figure 1: Illustration of an ALM stochastic Programming model

$$\text{Surplus Wealth} = \text{Assets} - \text{PV (liabilities)} - \text{PV (goals)}$$

Asset allocation decision making is a crucial part of a company's risk management system. Currently, the idea of asset-class investing is becoming more complex than the usual preconceived notion of just investing in equities, fixed income products or cash products. This is mainly due to the fact that hundreds of separate and distinct asset classes can be identified, and still more are flooding the markets. In addition these asset classes have different risk and return combinations and their correlations to the other products vary. A landmark study '*Determinants of Portfolio Performance*' published in the Financial Analysts Journal (July-August 1986) by *Brinson, Hood and Beebower* demonstrates how powerful asset allocation is. The investment results of 91 very large pension funds were examined to determine how and why their results differed. They reasoned on the premise that only four elements could contribute to investment results: investment policy, individual security selection, market timing and costs. By using a regression analysis, they attributed the contribution (or lack of it) to each of the four elements. Their conclusions were quite astonishing. They found out that the biggest single factor explaining performance was simply the investment policy (asset allocation) decision that determined how much a fund should hold in stocks, bonds or cash. Attempts at market timing most of the time amounted to a reduction in returns, and individual stock selection on average resulted in a reduction to the funds' returns. There was a wider variation in individual stock selection impact than in market timing, and a few managers were able to affect performance during the time period in a positive manner.

From this is the importance that of distinguishing between *strategic* and *tactical* asset allocation decisions can be seen. Broadly, it could be said that tactical asset allocation (TAA) begins where strategic asset allocation (SAA) ends. SAA decisions are based upon long-term expected returns and estimations of risk, which are formed from a

variety of factors, among which are past returns and volatilities, forecasts of long-term economic growth, and perhaps, assessments of political risk. But these allocations are formed infrequently, leaving the asset decision to drift in the intermediate term. On the other hand, TAA is designed to reposition the risk and return profile of the long-term strategic asset allocation in response to intermediate-term variations. There will be a reduction in those asset classes where risk has risen to abnormal levels, while exposure is increased in those asset classes likely to provide a more favourable return. This is done not by attempting to maintain a constant profile, but rather by evaluating the near-term relative risk and return characteristics of each of the underlying asset classes, and optimally shifting exposure away from asset classes showing uncharacteristic near-term weakness, and in the direction of those exhibiting much more promise in terms of returns.

Even the newspapers have run headlines on the debate of asset allocation. The LA Times (4/9/97) quoted the following “Academic studies have demonstrated that asset allocation among stocks, bonds and cash is the key to your portfolio’s performance over time- much more important than the individual securities you select”. Given this statement and the illustration of the study of the determinants of portfolio performance above, the idea of asset allocation and its importance starts to strike home. It is in this light that ALM has to come into play to make sure that the asset allocation decisions are optimal and try to smooth the cash flows of financial institutions.

An Optimisation Approach

Mathematical Programming (MP) is the generic name for optimisation models which are used in planning. MP is characterised by the use of an objective function which must be optimised and a set of linear or non linear equations or inequalities called constraints which must be satisfied. The objective function is introduced to obtain a desirable or in some sense the best solution. This is because in general there are many (often infinitely many) different ways the constraints can be satisfied. However, the MP models turn this into a problem of making the best decision in contrast to any feasible decision. ALM as described above represent the requirements and the constraints of the cash flow matching which can be achieved in (infinitely) many different ways. Through use of objectives and goals therefore we formulate optimisation models which lead to best ALM ‘matching’ decisions.

ALM models: Optimum Hedged Decisions under Uncertainty

In all real world planning problems in general, and in financial planning problems in particular, time and uncertainty play a key role. Thus optimum plans cannot be made in a deterministic way since the asset prices and the liabilities are not known with certainty in the future. Under these circumstances the concept of optimum plans is extended to optimum hedged plans. In order to achieve this, optimum allocation models are brought together with models of randomness which compute the possible future prices and liabilities (Figure 2). This combined paradigm of models is often known as the *stochastic optimisation* models. Using such SP models, it is possible to compute hedged decisions which may not be the best for any one realisation of the

future but is robust in respect of different realisation of the future. It is easily seen that a good description of uncertainty may significantly improve on ALM decisions.

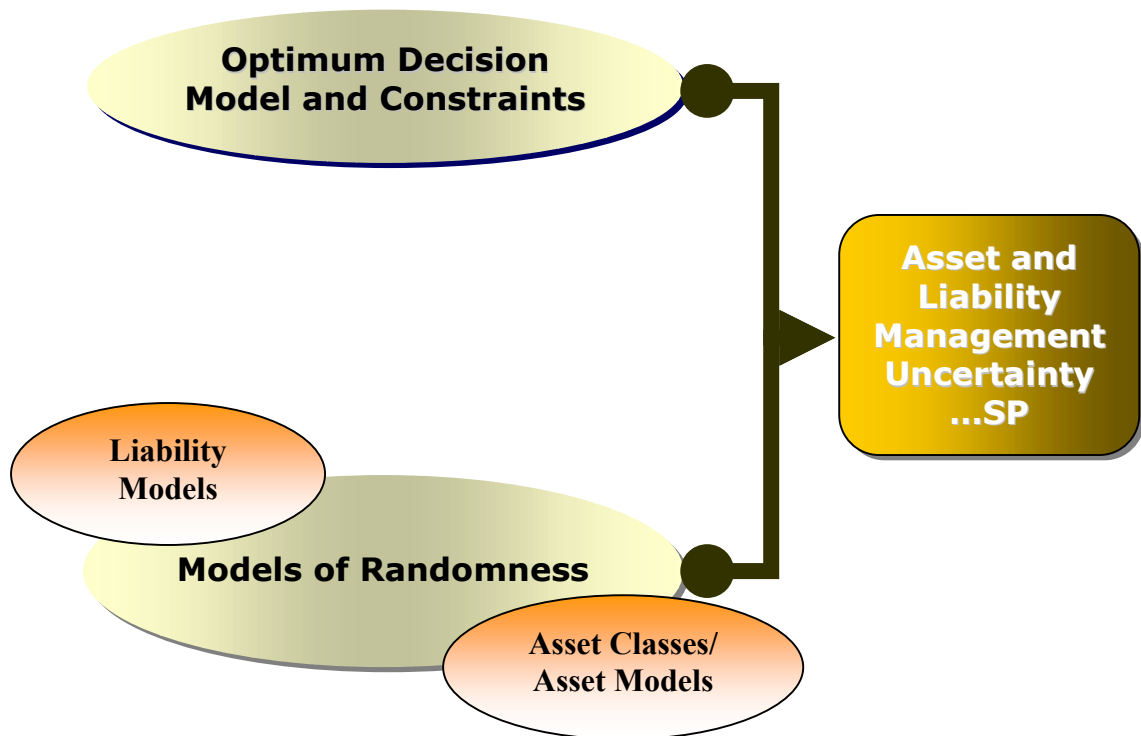


Figure 2

Pieter Klassen has made an interesting study entitled *“Solving Stochastic Programming Models for Asset/ Liability Management using Iterative Disaggregation”* , which provides some insight into the ALM techniques. In his paper, Klassen points out that a well-known problem that arises by using stochastic programming models in practice is that, only a limited amount of uncertainty can be included- because of the numerical optimisation methods that are used. While the description should be representative of the true uncertainty, it also should be the case that uncertainty which does not affect optimal decisions can be left out.

He views that to get a good description of the uncertainty in future asset prices and returns, they have to be free of arbitrage opportunities and consistent with market prices. Yet, when stochastic programming models for portfolio investment problems are formulated, these properties are set aside most of the time. He shows that a violation of these properties may lead to optimal portfolios in stochastic programming models which are severely biased towards spurious profit opportunities.

Future uncertainties are often captured using an event tree which is a simple but effective model of randomness. A diagrammatic illustration of an event tree structure (e.g taken in SPInE by Valente, Mitra, Poojari and Kyriakis) is depicted showing the possible future scenarios(Figure 3)

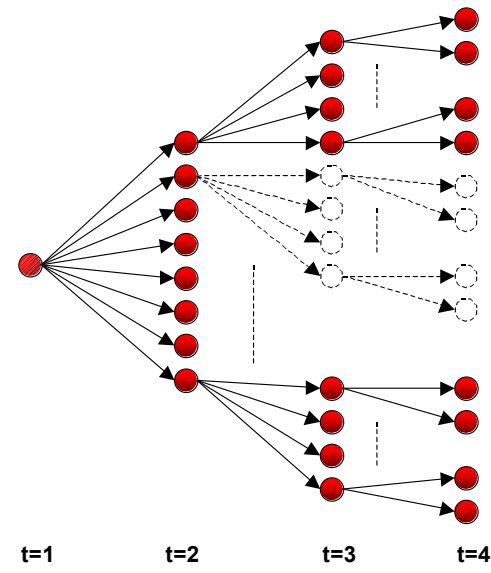


Figure 3: Event tree structure

In this event tree structure, there are 64 scenarios. In fact, there are 8 possible outcomes in the second stage, 4 conditional outcomes in the third stage and 2 conditional outcomes in the last stage- giving a total of $8*4*2=64$ scenarios. Another diagram shows how ALM and SP are integrated.

Risks faced by Financial Institutions & Risk Quantification

In the finance world, future unpredictability is termed volatility: volatility of asset prices and uncertain liabilities clearly affects financial plans. In general such uncertainties lead to possible financial loss or in other words financial risk. But that does not mean uncertainty equates to or is synonymous with risk. Depending upon the decision-maker or the fund manager's utility, there are many alternative measures of risk.

The choice of an appropriate risk measure that captures an individual's investment preferences has been, and continues to be, the subject of a long debate between academics and practitioners. This is not surprising, since without prior assumptions on the risk preferences of the individuals or the forms of the alternative distributions, it is likely that two individuals will consider risk from alternative perspectives.

In general, risk measures can be divided in two groups depending on their perception of risk. The first group contains the so-called dispersion risk measures that quantify

risk in terms of probability-weighted dispersion of results around a specific reference point, usually the expected value. Measures in this category penalize negative as well as positive deviations from a pre-specified target. Two of the most well-known and widely applied risk measures, in this group, are Markowitz's (1952, 1959) *variance* or *standard deviation* and the expected or *mean absolute deviation* of Adikson (1970) and Konno and Yamasaki (1991). The second group comprises measures which quantify risk according to results and probabilities below reference points, selected either subjectively or objectively. Such risk measures include the *Expected Value of Loss* from Domar and Musgrave (1944), Roy's (1952) *Safety First*, the *Semi-Variance* proposed by Markowitz (1959), and Fishburn's $\alpha - t$ criterion (1977) that not only constitutes the generalized case for the above 'below-target' risk measures. In 1993, this concept of loss beyond a target has taken considerable way, found by JP Morgan's analyst who introduced *Value at Risk* (JP Morgan, 1993) as a measure of loss to level beyond a given percentile of distribution.

Regulation of risk is naturally important in the context of banks and financial institutions' planning and operations. The Bank of International Settlements (BIS) started their work in Basle in 1988 (Basle Accord) and since then introduced regulatory requirements which are frequently updated (2000, new accord 2002). These regulations are globally followed by financial institutions. The risks faced by financial institutions come from different sources of uncertainty. These are then classified accordingly. Today the following are the accepted areas of risk- operational, credit, liquidity, operational, systemic, political and legal risks.

It is a widely accepted notion that financial institutions and more generally banks are in the business of managing risks. The better they manage these risks, the better placed they are in dealing with very rare but possibly commercially destructive events. Moreover, as it is known in the financial markets- 'a company's reputation is only as good as its last transaction'; hence any let-up in controlling the different aspects of a business could severely dent its future expansion.

The following are some of the financial risks that an organisation may encounter. By financial risks, broadly means that part of uncertainty that relates to the returns of assets arising from unanticipated and unpredictable events. These events may initiate runs on banks or create a banking panic. In this part, some of the more common risks are discussed. First of all, **credit risk** means the risk that arises in the event that a counterparty defaults on its obligations. The losses can be very substantial for any firm- for example, defaults on mortgage payments or companies' not honouring their bond repayments. Moreover, **liquidity risks** are defined as an event when it is difficult or expensive to make changes in the composition of one's portfolio. This usually takes place when there are crises in the global markets or following some unexpected political events.

Furthermore, **political risks** are usually country-specific and relate to the political uncertainties and policies of a particular government- a current example of the existence of political risks could be Zimbabwe where recent events have created some instability and reduced investment in the economy. Finally, we could define a situation where the financial sector has collapsed and where runs on banks are present and problems of liquidity and defaults surface- an 'apocalyptic' situation in a sense- as **systemic risk**.

Past, Present & Future: A decision-making Perspective

There are well-known models such as the Markowitz mean-variance model which has been used to capture uncertainty and make hedged decisions. Unfortunately, it relies entirely on history and makes a single period static decision. The real question that should be asked is: whether history should be taken into account to make future decisions? History doesn't always repeat. So the answer is that historical data should not be ignored. However, our models should be forward-looking with event trees of future scenarios. The flow of data and processing this into an analytic database (datamarts) and finally use of models which support hedged optimum decisions are shown in Figure 4.

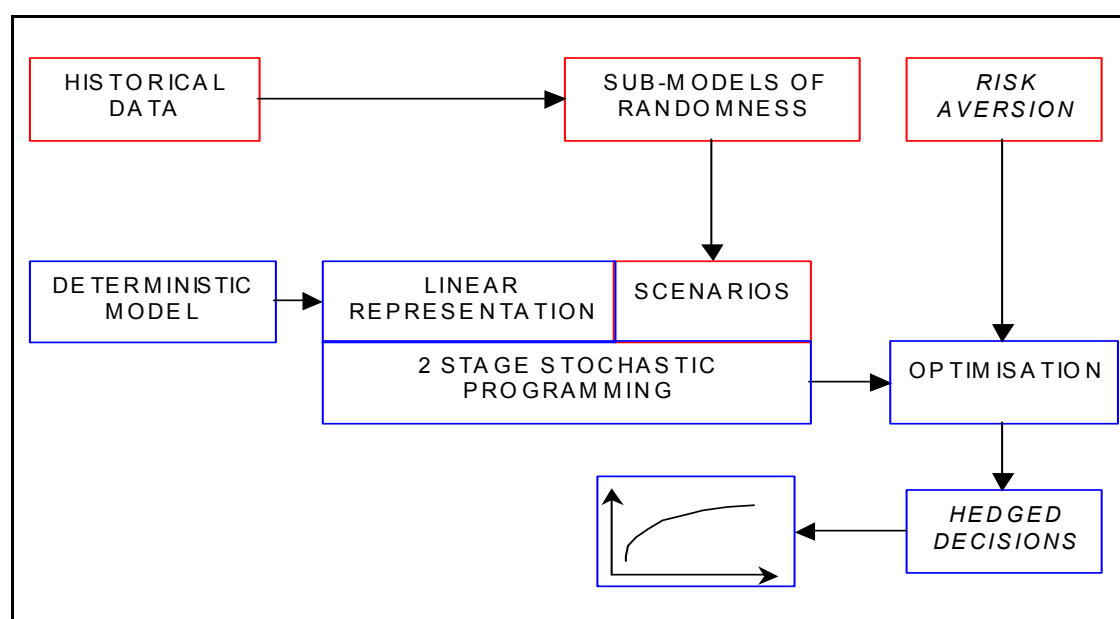


Figure 4: From Historical data to Optimum ALM decisions

Model Based-applications of ALM

The basic concepts of ALM models under uncertainty were developed by Kallberg, White & Ziemba (1982) and Kusy & Ziemba (1986). Afterwards, large scale applications were developed. They include the Russell-Yasuda Kasai model by Carino, Kent, Myers, Stacy, Sylvanus, Turner, Watanabe & Ziemba. This is an asset/liability model model for a Japanese insurance company using multistage stochastic programming. According to the authors, the model has enabled Yasuda to make use of a state-of-the-art decision-making and risk-management tool that provides valuable insights into the complex choices and restrictions to which the business is exposed.

Then there is the *computer-aided asset/ liability* management (CALM) stochastic programming model for dynamic ALM by Consigli & Dempster (1998). They present the CALM model which has been designed to deal with uncertainty affecting both

assets (in either the portfolio or the market) and liabilities (in the form of scenario dependent payments or borrowing costs). The discussion continues with the presentation of Stavros Zenios' paper on; 'ALM under uncertainty for Fixed Income Securities', (1995). His model made it possible to capture the increasing complexities of the fixed-income markets. He also found that the use of stochastic models is well justified given their superior performance over traditional immunization techniques.

Finally, there is, 'A dynamic model for ALM for defined benefit pension funds', by Cees Dert (1995). This paper presents a scenario-based optimisation model for analysing the investment policy and funding policy of pension funds, taking into account the development of the liabilities in conjunction with the economic environment. The ALM model presented can be used to compute ALM strategies which specify investment decisions and contribution levels to be set under a wide range of future circumstances. Dert found that decisions reached were different when using dynamic ALM strategies compared to static policies. Also, the use of the ALM model resulted in strategies with lower funding costs, the probabilities of under-funding were substantially smaller and the magnitude of deficits, reflected by the costs of remedial contributions, has been reduced dramatically.

Given that there are various complex financial products being traded in the markets and over-the-counter, not only has it become much harder to assess the potential risks of some of these stand-alone products but also the problem of integrating these risks in the risk management system has arisen. Some research has been done on this topic by *Martin Holmer on "Integrated Asset-Liability Management: An Implementation Case Study"*. His articles looks at integrated ALM which he views as a new management perspective that is creeping its way into the more inventive financial intermediaries in reaction to problems inherited from the older functional management perspective. For the latter, an organization has to be structured into different functional units (e.g., marketing, asset management, etc.), the decisions of which are synchronized by a corporate plan based on macroeconomic forecast. However, the lack of precision in predicting macroeconomic variables has forced the hands of some banks in looking for alternative management perspectives.

Hence the new concept of integrated ALM. This perspective, as its name implies, is more focused on integrating the various units of the organisation in order to include all the functional activity related to a line of business. Decisions are taken with the help of computer models, also trying to ascertain the uncertainty of the future business environment, and to generate profitable strategies by structuring the assets and liabilities of the business line across a series of alternative future scenarios. By comparing these alternative ways, a crucial difference can be spotted which is that, decisions are made using profitability calculations based on a single-scenario planning forecast, while, for the integrated ALM, decisions are made using risk-adjusted or hedged profitability calculations based on multiple-scenario possibilities. This is just a brief synopsis of how ALM can be used in the decision-making process.

What is available on the market

With new breakthroughs in technology, it was no surprise researchers tried to apply their findings in ways that are useful to institutional and private investors. In fact,

some interesting software were created to cater the need of this niche market. In this section, some of them will be discussed.

First of all, there is **PROFITstar**, which is an asset-liability management, budgeting and simulation application that can help in decision-making for institutions. It uses an integrated, strategic approach to managing financial goals and the position of institutions. Some of the other interesting features are that it helps to avoid inaccuracies that can result from relying solely on contractual maturity roll-off. Moreover, **PROFITstar**'s Interest Rate Sensitivity (IRSA) Matrix is able to simulate eight distinct rate swings and analyse their effects on income, capital and other key ratios.

The following software provider **Surya** is one that has developed products such as **MitiGet** (a market risk control system) and **Balm** (a bank asset liability management system). The following are some of the features of **MitiGet**:

- Tracks portfolio by exposure, stand-alone performance and relative performance
- Tracks MTM and period returns
- Has a comprehensive cash flow definition module, modeling all dimensions of cash flow as random variables
- Facilitates analysis of position, return and market risk on multiple and flexible dimensions such as industry, region, credit rating, market cap and liquidity
- Facilitates dynamic grouping
- Supports risk policy definition in terms of limits on both positions and value at risk
- Supports two limits - soft and hard limits
- Supports hierarchical and 'across the board' concentration groups for limit definitions
- Sensitivity and Scenario Analysis

Concerning **Balm**, it facilitates measurements of:

- 1) Structural liquidity, and
- 2) Interest rate sensitivity.

The *liquidity* module features of **Balm** are:

- Ability to compare cash flow mismatches across maturity spectrum,
- Can set tolerance limit for mismatch,
- The highlighting of maturity buckets that are under stress,
- The comparison of cash flow mismatches against tolerance limit.

Following are the features of the interest rate sensitivity module:

- It segregates assets and liabilities into interest rate sensitive and non-interest rate sensitive categories. Further, interest rate sensitive assets and liabilities are sliced and distributed across seven maturity buckets.
- Maturity buckets that have mismatch between rate sensitive assets and rate sensitive liabilities are highlighted.
- Identifies components of assets and liabilities that contribute to mismatch in a given maturity bucket. This enables the user to initiate corrective action.

- Users can generate several interest rate scenarios across maturity buckets and evaluate their impact on Net Interest Income (NII).

Stochastic Programming Integrated Environment (SPInE) software. The prototype of **SPInE** was initially designed by Messina and Mitra in 1997. Subsequently, there was a revision of the first **SPInE** prototype; where several features were added, such as the support of the SMPL and SAMPL extended languages. By combining natural definitions of the randomness of the problems with the existing features of these optimisation systems, such extensions introduce powerful constructs for formulating complex stochastic programming and chance constrained programming models. The modelling subsystem is able to generate model data in the SMPS format representation, giving SPInE the ability to link any external solver which supports this standard.

Closely coupled with the modelling system, SPInE embeds a stochastic solver which incorporates alternative solution algorithms including:

- Benders' decomposition
- Lagrangean relaxation

The solver is also capable of computing good discrete feasible solutions to “real world” instances of mixed integer SP models. Deterministic equivalent instances may be also constructed and solved using the Interior Point Method (IPM).

SPInE is sufficiently versatile and allows the modeller to perform scenario analysis, analysis of 'Here and Now' and the 'expected value' solutions. Stochastic information such as the expected value of perfect information (EVPI) and the value of stochastic solution (VSS) are easily computed. By supporting the ODBC standard for database connection, commercial systems can be used to link SPInE with scenario generators as well as to store and analyse the application data. The user can also take advantage of multidimensional data viewers, like On-Line Analytical Processing (OLAP) tools, for the analysis of the model data and the corresponding (optimal) solutions.

The modular architecture of SPInE makes it easy to embed the various systems' components into customized applications; taken together, the components comprise a flexible platform for building vertical solutions.

Concluding Remarks

In this white paper, important recent developments and the pragmatic approaches to the ALM models and systems have been presented. An explanation to show how ALM is becoming the linchpin of firms' financial management has been given, especially under conditions of uncertainty which requires risk management. An illustration of how ALM and SP is integrated using an optimisation and risk control paradigm have also been discussed.

So where does ALM take us in the future? Well, modelling the true uncertainty may still be an area under investigation. Moreover, the ability to devise software that can

add more features would be most welcome. For example, presently it is not possible to incorporate TAA in ALM.

SP has proved to be a powerful modelling approach to optimum decision-making under uncertainty. It has been shown to be more appropriate in many applications. Improvements in the new technologies and solution methods have made SP a viable optimisation tool, especially in the domain of asset and liability management.

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