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MEASURING PENSION FUND PERFORMANCE USING RISK-ADJUSTED MEASUREMENTS

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Measuring Pension Fund Performance using Risk-Adjusted Measurements

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1 Introduction and Background

This report uses risk-adjusted performance measures to identify the quality of alternative decision models. This report uses the decision models introduced by Schwaiger et al. (2007) and the simulation and decision evaluation outcomes from Schwaiger et al. (2008).

Risk-adjusted performance measures are used by fund managers to rank and compare their portfolio performance with peers. In this report we examine the performance of alternative decision models for pension funds and use two ratios, namely the Sortino and the Solvency ratio to measure their performance over time.

1.1 Background

In Schwaiger et al. (2007) five different approaches in tackling the pension fund problem are introduced: a linear deterministic model, a stochastic programming model, a chance-constrained programming model, an integrated chance-constrained programming model and a simple fixed mix model. This section reviews the deterministic and stochastic models, whose values of the decision variables from the optimization outcomes are then used to conduct simulation and decision evaluation.

In the deterministic model the model indices are b , which is an element of the total set of bonds available and t , which is the current time. The objective is to minimize the total deviations between the bonds and liabilities $PV01$ values and at the same time minimise initial injected cash. The decision variables are:

The model indices are:

b	bond from a bond set B
t	time horizon $1, \dots, T$

Secondly, we define the parameters:

L_t	liability value at time t
$LPV01_t$	PV01 value of the liability at time t
$BPV01_t^b$	PV01 value of bond b at time t
c_t^b	cash flow stream of bond b at time t
P_b	trading price of bond b
α	transaction cost involved when bonds are sold or bought
O_b	opening position of bond b at the initial time period
r_t	interest rate at time t

Now we define the decision variables. All variables are constrained to be nonnegative.

x_b	amount of bond b purchased
y_b	amount of bond b sold
z_b	amount of bond b held
d_t	amount borrowed at time period t ; limited to be $\leq M_t \forall t$
s_t	amount lent at time period t
C	initial available cash

Furthermore, we use the following decision variables to measure the deviations of the assets $PV01$

$devo_t$	measurement of over deviation at time t
$devu_t$	measurement of under deviation at time t

The set of constraints include some basic constraints for portfolio optimization: these are the cash-flow accounting equation and the inventory balance constraints. The cash-flow accounting equation initializes the new asset portfolio by liquidating the old asset allocation and including the available cash C .

$$\sum_{b=1}^B (1 + \alpha)P_b x_b \leq C + \sum_{b=1}^B (1 - \alpha)P_b y_b \quad (1)$$

The inventory balance constraint assures that the holding of bond b is made up of the opening position of the same bond including the amount bought minus the amount sold:

$$z_b = O_b + x_b - y_b \quad \forall b \quad (2)$$

The following constraints match the bonds cash flows with the liabilities stream, having the possibility to borrow from the bank or reinvest spare cash. At the final time period there is no opportunity to borrow from the bank anymore.

$$\sum_{b=1}^B c_1^b z_b + d_1 = L_1 + s_1 \quad (3)$$

$$\sum_{b=1}^B c_t^b z_b + d_t - (1 + r_t)d_{t-1} = L_t + s_t - (1 + r_t)s_{t-1} \quad t : 2 \leq t \leq T \quad (4)$$

$$\sum_{b=1}^B c_T^b z_b - (1 + r_T)d_{T-1} = L_T + s_T - (1 + r_T)s_{T-1} \quad (5)$$

Furthermore, we define the behavior of the variables $devo_t$ and $devu_t$ with the following constraint that binds them to the difference of the assets and liabilities $PV01$ values:

$$\sum_{b=1}^B BPV01_t^b z_b = devo_t - devu_t + LPV01_t \quad \forall t \quad (6)$$

If we include the minimum and maximum amount of bonds in the portfolio within a specific rating class, say AAA rating, specified by the pension fund, we get the

following user specific constraints:

$$MinInRatingclassAAA \leq \sum_{bonds} z_t^{b(i=AAA,\dots)} \leq MaxInRatingclassAAA \quad \forall t \quad (7)$$

Similar limits are imposed in respect of other bond characteristic, that is country, issuer and sector.

An increased need of initial cash means an increase in the active members' contributions. The first objective function is the initial cash, which has to be injected in advance to achieve a (feasible) matching between assets and liabilities:

$$\phi_1(d_t) = Minimize C + \sum_{t=1}^T d_t \quad (8)$$

The second objective function is the total deviations of assets and liabilities; there are other suitable objective functions we might consider depending on the decision values required.

The objective function minimizes the total *PV01* deviations of assets and liabilities. This linear objective function is given by

$$\phi_2(devo_t, devu_t) = Minimize \sum_{t=1}^T (devo_t + devu_t). \quad (9)$$

In the two-stage stochastic programming models (including the chance-constrained and integrated chance-constrained programming models) the uncertainty is represented in interest rate scenarios. The objective is to minimize the total present value deviations of assets and liabilities and at the same time minimizing initial injected cash.

The sets:

b	bond from a bond set B
t	time horizon $1, \dots, T$
s	scenario $1, \dots, S$

The parameters:

c_t^b	cash flow stream of bond b at time t
P_b	trading price of bond b
α	bid-ask spread in percentage when bonds are sold or bought
O_b	opening position of bond b at the initial time period
γ_t	weight of liabilities with respect to the asset value at time t ; likely to be > 1 can be user defined with high values at the earlier time points and then decreasing with time
β_t	reliability level at time t ; likely to be > 0 ; can be user defined or assumed to be constant over time
λ_t	maximum expected shortfall in fraction of the liabilities at time t ; again user defined or assumed to be constant
N	sufficiently large number for the chance constraint (maximum value the investment portfolio is likely to reach)

The stochastic parameters:

L_t^s	liability value at time t under scenario s
r_t^s	uncertain interest rate at time t
π_s	probability of scenario s occurring, equiprobable
LPV_t^s	uncertain present value of the liability at time t
$BPV_t^{b,s}$	uncertain present value of bond b at time t

The implementable first stage decision variables:

x_b	amount of bond b purchased
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y_b	amount of bond b sold
z_b	amount of bond b held
C	initial cash

The following are the non-implementable stochastic decision variables:

d_t^s	uncertain amount borrowed at time period t ; limited to be $\leq M_t \forall t$
s_t^s	uncertain amount lent at time period t
$shortage_t^s$	amount of underfunding at time t
δ_t^s	binary variable, which takes the value of 1 if there is any underfunding

The constraints:

cash-flow accounting equation:

$$\sum_{b=1}^B (1 + \alpha) P_b x_b \leq C + \sum_{b=1}^B (1 - \alpha) P_b y_b \quad (10)$$

Inventory balance equation:

$$z_b = O_b + x_b - y_b \quad \forall b \quad (11)$$

Matching equations for all time periods:

$$\sum_{b=1}^B c_1^b z_b + d_1^s = L_1^s + s_1^s \quad \forall s \quad (12)$$

$$\sum_{b=1}^B c_t^b z_b + d_t^s - (1 + r_t) d_{t-1}^s = L_t^s + s_t^s - (1 + r_t) s_{t-1}^s \quad \forall s, t \geq 2 \quad (13)$$

$$\sum_{b=1}^B c_T^b z_b - (1 + r_T) d_{T-1}^s = L_T^s + s_T^s - (1 + r_T) s_{T-1}^s \quad \forall s \quad (14)$$

Non-Anticipativity constraints:

$$d_1^s = d_1^1 \quad s = 2, \dots, S \quad (15)$$

$$s_1^s = d_1^1 \quad s = 2, \dots, S \quad (16)$$

The chance constraints are formulated as follows:

$$P\left\{\sum_{b=1}^B BPV_{b,t+1}^s z_b + PV s_{t+1}^s - \gamma LPV_{t+1}^s - \gamma PV d_{t+1}^s \geq 0 \mid (t, s)\right\} \geq \beta_t \quad \forall t \quad (17)$$

$$N \delta_{t+1}^s \geq \gamma LPV_{t+1}^s + \gamma PV d_{t+1}^s - \sum_{b=1}^B BPV_{b,t+1}^s z_b - PV s_{t+1}^s, \quad (18)$$

$$\pi_s \sum_{s=1}^S \delta_{t+1}^s \leq 1 - \beta_t \quad \forall t \quad (19)$$

$$N(1 - \delta_{t+1}^s) - \frac{1}{N} \geq BPV_{b,t+1}^s z_b + PV s_{t+1}^s - \gamma LPV_{t+1}^s - \gamma PV d_{t+1}^s \quad \forall t, s \quad (20)$$

$$\delta_t^s \in \{0, 1\} \quad \forall t \quad (21)$$

The integrated chance constraints can be included in the following way:

$$\sum_{b=1}^B BPV_{b,t}^s z_b + PV s_t^s - \gamma LPV_t^s - \gamma PV d_t^s + shortage_t^s \geq 0 \quad \forall t, s \quad (22)$$

$$\pi \sum_{s=1}^S shortage_t^s \leq \lambda \max_s(LP V_t^s) \quad \forall t \quad (23)$$

The objective functions are:

$$\phi_5(d_t^s) = \text{Minimize } C + \sum_{t=1}^T \sum_{s=1}^S \pi_s(d_t^s) \quad (24)$$

and:

$$\phi_6(\text{devo}_t^s, \text{devu}_t^s) = \text{Minimize } \sum_{t=1}^T \sum_{s=1}^S \pi_s(\text{devo}_t^s + \text{devu}_t^s) \quad (25)$$

2 Risk-Adjusted Performance Measures

The performance of any fund can be measured using different measurement. We introduce the most used ones: from academia: (1) the standard deviation of returns, (2) the Sharpe ratio (see also Sharpe, 1966 and 1994), (3) Value at Risk, (4) the Jensen Index and from industry practice: (5) the Sortino ratio, (6) the Treynor ratio, (7) the Modigliani or M-Square Measure (see Modigliani and Modigliani 1997) and (8) the Morningstar Rating:

1. As financial interpretation, the standard deviation of a fund represents its risk. Fund Managers look at the standard deviation of excess return over the risk free rate or some benchmark. The standard deviation of the difference between the funds return and a benchmark return is also called the tracking error. The higher the standard deviation, the higher the expected return for an investment and the more risk bearing the investor.

The **standard deviation** can be calculated the following way:

$$\sigma = \sqrt{\frac{1}{N} \sum_{i=1}^N (x_i - \bar{x})^2} \quad (26)$$

where \bar{x} is the mean of return and N is the number of observations.

2. The Sharpe ratio (or Sharpe index) measures the excess return per unit of risk in the investment. All models have the same benchmark and the fund manager will choose the investment with the highest Sharpe ratio.

The **ex ante Sharpe ratio** is given as follows:

$$S \equiv \frac{\bar{d}}{\sigma_d}, \quad (27)$$

where \bar{d} is the expected value of the differential return of the fund and a benchmark, i.e.

$$\bar{d} \equiv E[R_P - R_I]$$

and σ_d is the predicted standard deviation of \tilde{d} , which is the forthcoming differential return.

The **ex post Sharpe ratio** is as follows:

$$S_h \equiv \sqrt{\frac{\bar{D}}{\sigma_D}}, \quad (28)$$

where

$$\begin{aligned} D_t &\equiv R_{Pt} - R_{It} \\ \bar{D} &\equiv \frac{1}{T} \sum_{t=1}^T D_t \\ \sigma_D &\equiv \sqrt{\frac{\sum_{t=1}^T (D_t - \bar{D})^2}{T - 1}} \end{aligned}$$

3. Value at Risk (VaR) measures the least expected loss (relative to wealth) that will be expected given a certain possibility (confidence level).

Value at Risk can be calculated as follows (see Gaivoronski 2005):

$$VaR(W) = \mathbb{E}(W) - \inf\{u : F(u) > \alpha\} \quad (29)$$

where α is the quantile of return and $1 - \alpha$ is the confidence level.

4. The Jensen Index measures the performance against the market index. A high Jensen Index is interpreted as higher return given a risk level on the portfolio.

The **Jensen Index** is given by (see Jensen 1969):

$$J = R_P - r_f - \beta(R_I - r_f) \quad (30)$$

The Treynor ratio T is given by:

$$T = \frac{R_P - r_f}{\beta}, \quad (31)$$

where R_P is the portfolio return, r_f is the risk free rate and $\beta = \frac{Cov(R_P, R_M)}{Var(R_M)}$ is the portfolio β with respect to the market.

5. The Sortino Ratio roots from the Sortino Ratio, but it is industry widely used since it only penalizes a portfolios underperformance via the downside deviation.

The **Sortino ratio** is calculated by:

$$S \equiv \frac{R_P - R_I}{\sigma_d}, \quad (32)$$

where

$$R_P \equiv \left(\prod_{i=1}^N (1 + R_{P_i}) \right)^{\frac{1}{N}} - 1$$

$$\sigma_d \equiv \sqrt{\frac{1}{N} \sum_{i=1}^N (\min(R_{P_i} - R_I, 0))^2}$$

The term σ_d is also called the downside deviation or target semideviation.

6. The Treynor ratio measures the returns earned in excess of which could be earned on a riskless investment. The higher the Treynor ratio, the better the performance of the fund strategy. Like the Sharpe ratio, the Treynor ratio is based on the capital asset pricing model (CAPM), which means that any investor can achieve any level of risk by investing in the fund with the highest Sharpe ratio regardless of his/her degree of risk aversion.

7. Modigliani et al. propose an alternative measure which is easy to interpret as a fund manager. The Modigliani measure evaluates in basis points, how much a portfolio out- or underperformed the market. The higher the Modigliani Measure, the higher the fund's return for any level of risk.

The **Modigliani** or **M-Square Measure** (see Modigliani and Modigliani 1997) is given as the following:

$$RAPA(F) = \frac{\sigma_I}{\sigma_F} \bar{e}_F \quad (33)$$

where RAPA stands for Risk-adjusted performance, σ_I and σ_F are the standard deviation of the Benchmark excess return and the Fund's excess return, respectively, and \bar{e}_F is the Fund's average excess return.

8. Fund Managers often use the star rating or Morningstar Rating from Morningstar Inc. which classifies funds into five ratings. To obtain these rating

they use their own risk return measures.

For the **Morningstar Ratings**, the excess return of a bond fund is calculated adjusting for sales loads and subtracting the 90 day Treasury bill rate.

$$\text{Morningstar Return} = \frac{\text{SalesLoadAdjustedExcessReturn} - 90\text{dayTreasuryBillReturn}}{\text{AverageExcessReturnforBondFundClass}} \quad (34)$$

The fund return is adjusted for maximum front-end loads, applicable deferred loads and applicable redemption fees. The top 10% of the funds receive a 5 star Rating, the next 22.5% receive 4 Stars, next 35% 3 Stars, next 22.5% 2 Stars and the last 10% 1 Star.

$$\text{Morningstar Risk} = \frac{\text{Fund's Average Underperformance}}{\text{Average Underperformance of Bond Fund Class}} \quad (35)$$

An additional method to evaluate the performance of the different models is looking at the solvency ratio:

$$\text{Solvency} = \frac{\text{wealth} - \text{technical reserves}}{\text{technical reserves}} \quad (36)$$

Assumptions about Input Variables

For

calculating the Solvency and the Sortino ratio, the following assumptions are made:

Variable	Used Input	Description/Explanation
<i>wealth</i>	Total Inflows	Portfolio asset wealth and saved cash
<i>technical reserves</i>	Total Outflows	Liability Value and amount borrowed
R_P	Portfolio Return	Portfolio return generated from assets and cash
R_I	Liability Return	Liabilities are meant to be matched, which makes them the benchmark to be met

The aim of the models is to match the liabilities stream with a bond portfolio, where over- and underdeviations are penalized equally. From the ratios we aim to have *inflows = outflows*, which makes the numerator in the solvency ratio equal to zero and the same in the Sortino ratio. For matching strategies both ratios reflect good results if they are close to zero. For the further sections only the Sortino and the Solvency ratios are used.

3 Hypotheses Testing

Hypothesis testing is used to decide statistically about the computational results of this work. There are five steps involved: (a) Identifying the null hypothesis: $H_0 : \mu_X = y$, where y is the value to be tested for. The test is used to find out if the mean of the Sortino and Solvency ratios are equal to zero, where μ_X is the mean of the Sortino and Solvency Ratio of the model X with X being the SP, CCP or ICCP model. The three possible alternative hypothesis and their critical region, where we reject H_0 are:

$$H_1 : \mu_X > y \quad t = \frac{\bar{X} - \mu_0}{S/\sqrt{n}} > t_{\alpha,d.f.} \quad (37)$$

$$H_1 : \mu_X < y \quad t = \frac{\bar{X} - \mu_0}{S/\sqrt{n}} < -t_{\alpha,d.f.} \quad (38)$$

$$H_1 : \mu_X \neq y \quad |t| = \frac{\bar{X} - \mu_0}{S/\sqrt{n}} > t_{\alpha/2,d.f.} \quad (39)$$

The second step (b) is to choose a rejection level α , which is normally 1% or 5%. Then (c) we select a test statistic (t-test in this case) and calculate its value from the sample data, which is called the observed value of the test statistic. We then (d) compare the observed value to the critical value obtained for the α level. And the last step (e) is to make a decision if either to accept the null hypothesis or if to reject it and accept the alternate hypothesis.

4 Computational Investigation

Two ratios are used to measure the portfolios performance: the Solvency ratio and the Sortino ratio. The solvency ratio is easily computed and widely understood. The Sortino ratio is an extension of the Sharpe ratio, where as a measure of risk the semi-standard deviation is used. The semi-variance only uses the probability of shortfall, unlike variance, which accounts for positive and negative outcomes

equally. The following graph (Figure 1) shows the Solvency and the Sortino ratios for the SP, CCP and ICCP models. The higher the ratios the better, but in case of matching, where the liabilities are the benchmark and outperformance is penalized, both ratios reflect good results when they are close to zero.

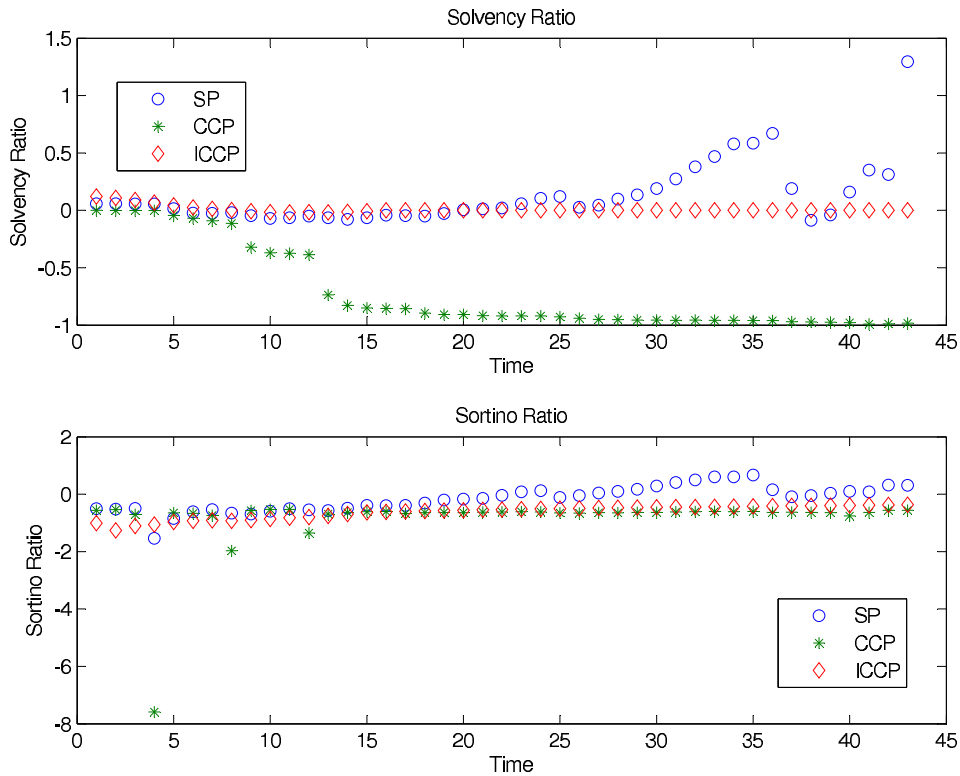


Figure 1: Solvency and Sortino Ratios for the SP, CCP and ICCP Model

From Figure 1 it can be seen that the SP model has a solvency ratio close to zero for half the time periods and then it increases in the other half, the CCP has a downward sloping solvency ratio, which means that the liabilities are growing faster than the assets and finally, the ICCP model has a very smooth solvency ratio close to zero for all time periods, which means that the assets match the liabilities nearly perfectly. The second graph explains a bit more: the SP model has an upward sloping Sortino ratio, which means that above zero the risk of making big losses gets smaller and that the assets outperform the liabilities, the

CCP model has at some points in time a high risk of making a big loss when looking at its Sortino ratio, while again the ICCP is close to zero which means it is closely matching the liabilities.

Hypothesis Testing

The models are further analyzed using hypothesis testing, where there are three different null hypothesis:

$$H_0 : \mu_{ICCP} = 0 \quad (40)$$

$$\mu_{CCP} = 0 \quad (41)$$

$$\mu_{SP} = 0 \quad (42)$$

and the corresponding alternate hypothesis:

$$H_1 : \mu_{ICCP} \neq 0 \quad (43)$$

$$\mu_{CCP} \neq 0 \quad (44)$$

$$\mu_{SP} \neq 0 \quad (45)$$

Table 1 shows the t-test statistics for the three null hypothesis for each ratio. For the CCP Solvency and CCP and ICCP Sortino ratio there is no evidence at all to accept the null hypothesis, while at 0.3%, 2.5% and 7% (2-tailed) we accept the null hypothesis for the SP Solvency, Sortino and ICCP Solvency ratio to have a mean of zero.

Stress Testing High/Low Interest Rates

The next set of graphs show the surplus/deficit frequency distribution at four different time periods: $t = 2, t = 10, t = 20$ and $t = 40$ for all four models. The following results (Figure 2 and Figure 3) show the surplus/deficit frequency distribution of the SP model using high and low interest rate scenarios. The SP model performs relatively well (in matching terms) compared to the other models and in both cases of high and low interest rate scenarios.

Ratios T-Test

One-Sample Statistics

	N	Mean	Std. Deviation	Std. Error Mean
SPSolvency	43	.12819075	.265937268	.04055505
SPSortino	43	-.15945969	.448835675	.06844680
CCPSortino	43	-.83592058	1.079673797	.16464871
CCPSolvency	43	-.71103581	.367624693	.05606224
ICCPSortino	43	-.62534591	.237841243	.03627045
ICCPSolveny	43	.00874622	.030838497	.00470283

One-Sample Test

	Test Value = 0					
	t	df	Sig. (2-tailed)	Mean Difference	95% Confidence Interval of the Difference	
					Lower	Upper
SPSolvency	3.161	42	.003	.128190747	.04634734	.21003416
SPSortino	-2.330	42	.025	-.159459689	-.29759092	-.02132846
CCPSortino	-5.077	42	.000	-.835920576	-1.168195	-.50364603
CCPSolvency	-12.683	42	.000	-.711035811	-.82417399	-.59789763
ICCPSortino	-17.241	42	.000	-.625345907	-.69854264	-.55214918
ICCPSolveny	1.860	42	.070	.008746221	-.00074447	.01823691

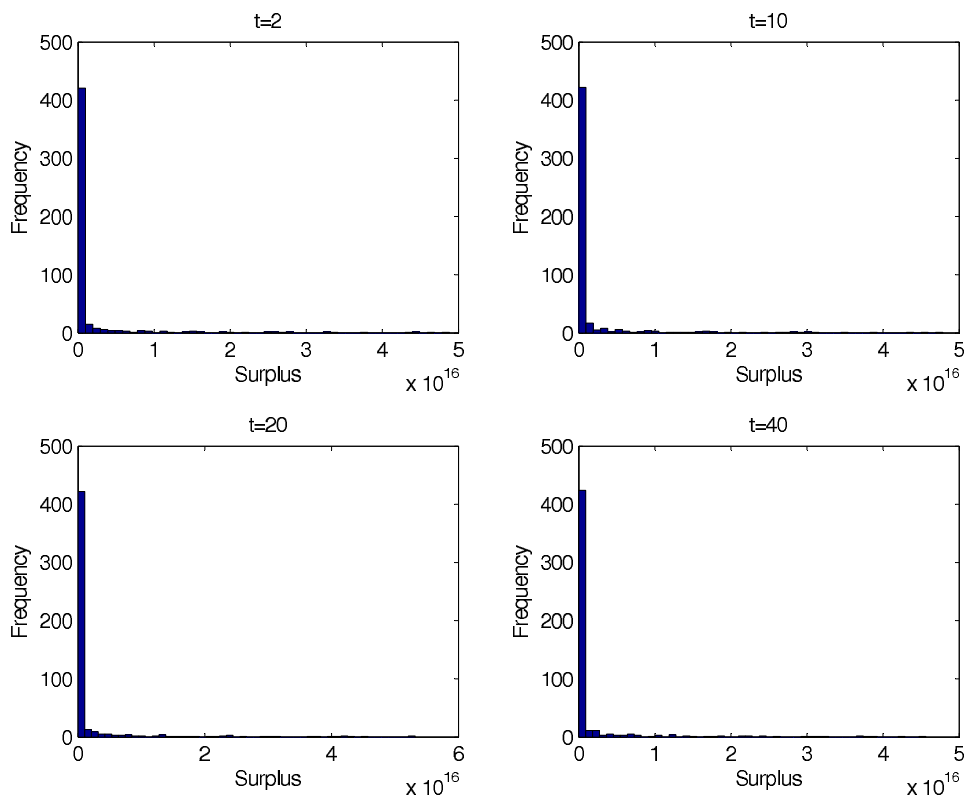


Figure 2: SP Model Stress Testing High Interest Rate Surplus/Deficit Distribution

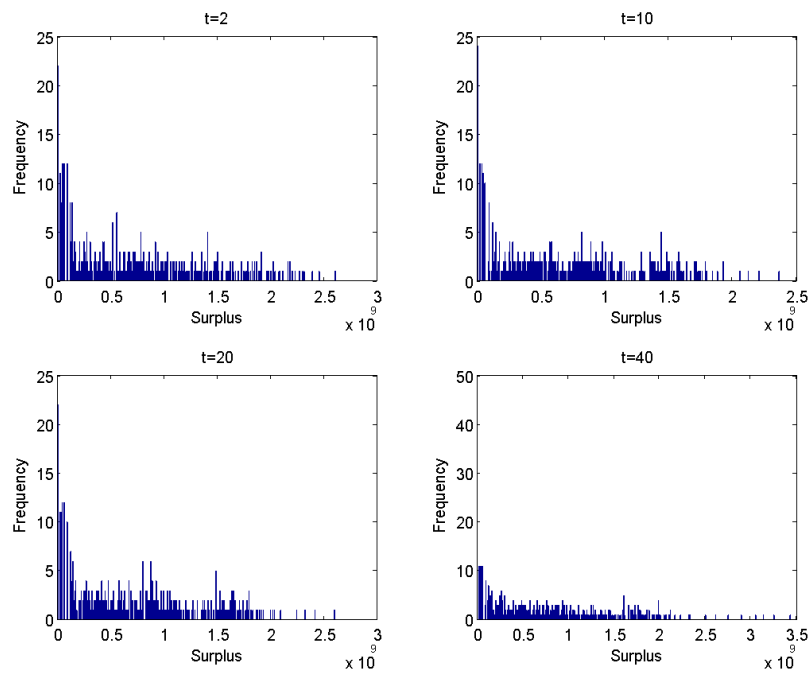


Figure 3: SP Model Stress Testing Low Interest Rate Surplus/Deficit Distribution

The following results (Figure 4 and Figure 5) show the surplus/deficit frequency distribution of the CCP model using high and low interest rate scenarios. In both cases the CCP generates a good asset/liability matching outcome. It can match the asset and liability present value deviations no matter what happens to the interest rates, this is due to the restriction of underfunding events which was incorporated into the model.

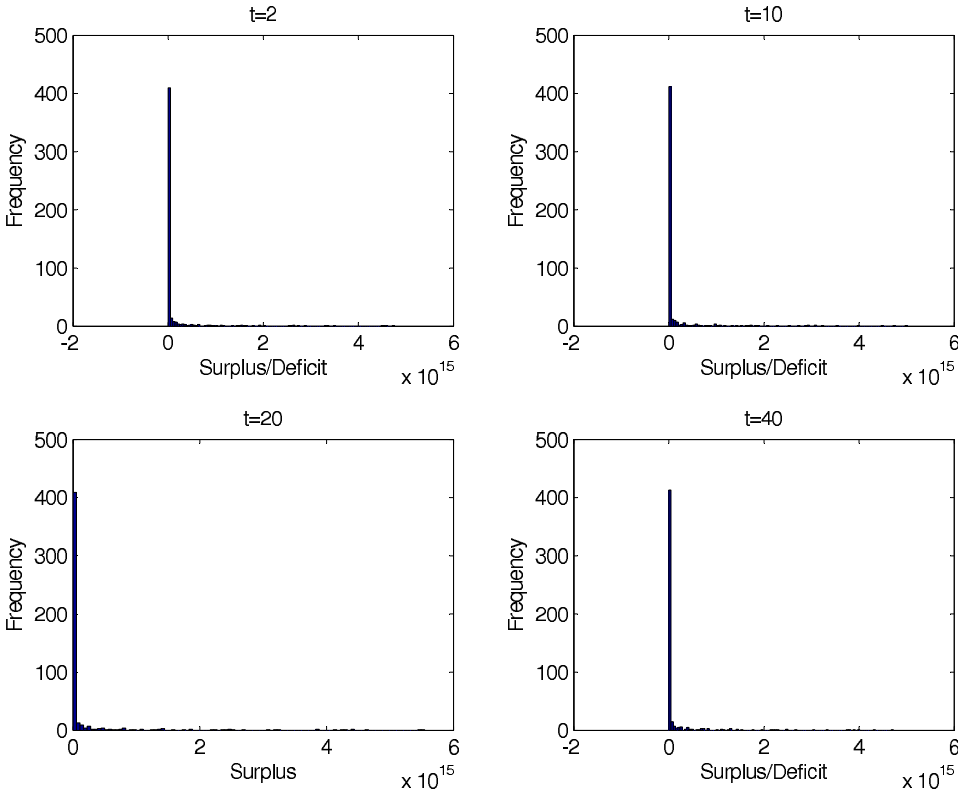


Figure 4: CCP Model Stress Testing High Interest Rate Surplus/Deficit Distribution

Figure 6 and Figure 7 show the surplus/deficit frequency distribution of the ICCP model using high and low interest rate scenarios. In low interest rate times the ICCP model guarantees fairly well a perfect matching, while in high interest rates

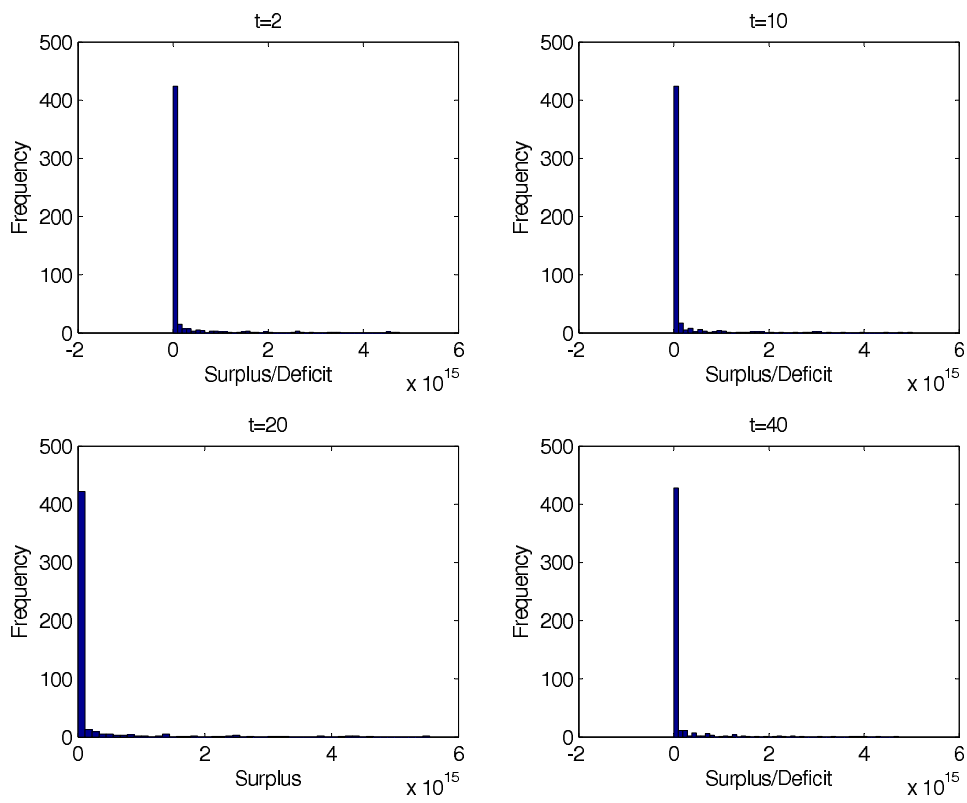


Figure 5: CCP Model Stress Testing Low Interest Rate Surplus/Deficit Distribution

the chance of having a loss is quite high but also some scenarios of high interest rates generate a high surplus. In low interest rate events, the discounted liabilities are higher than in high interest events; when liabilities are higher valued the ICCP model protects the pension fund from interest rate risk and generating a deficit. In high interest events the liabilities present values are lower but also the asset values, which then cannot cover for the outflows, leading to a deficit.

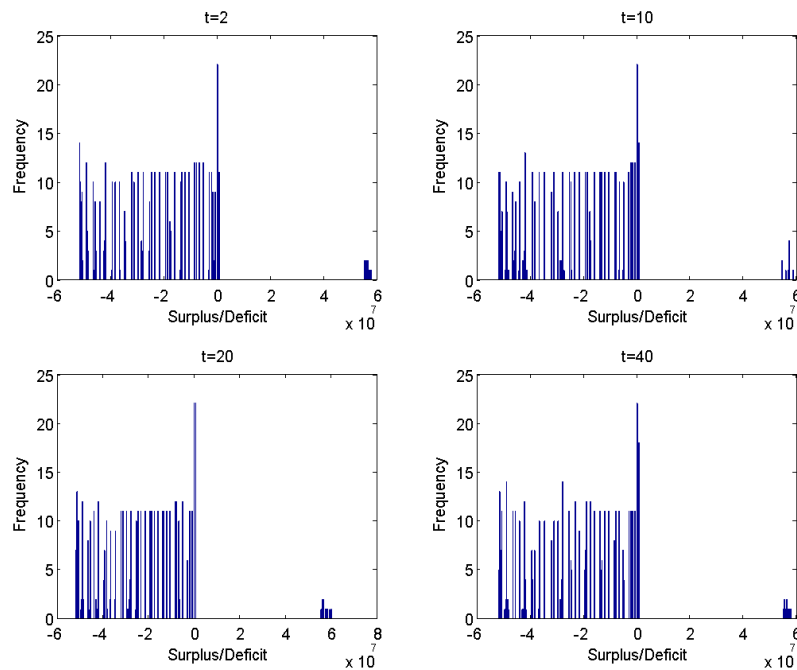


Figure 6: ICCP Model Stress Testing High Interest Rate Surplus/Deficit Distribution

The last results (Figure 8 and Figure 9) show the surplus/deficit frequency distribution of the LP model using high and low interest rate scenarios. As expected the LP model underperforms far more than all the other models in low interest rate scenarios. During high interest rates the LP model still does not perform better than any other model on average.

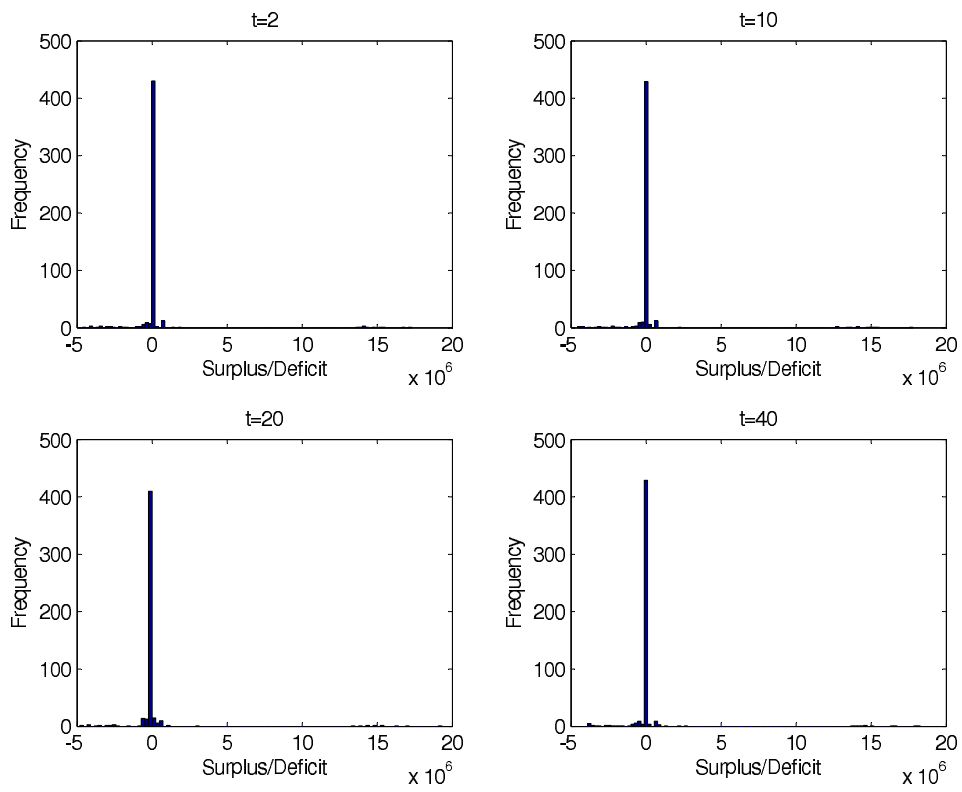


Figure 7: ICCP Model Stress Testing Low Interest Rate Surplus/Deficit Distribution

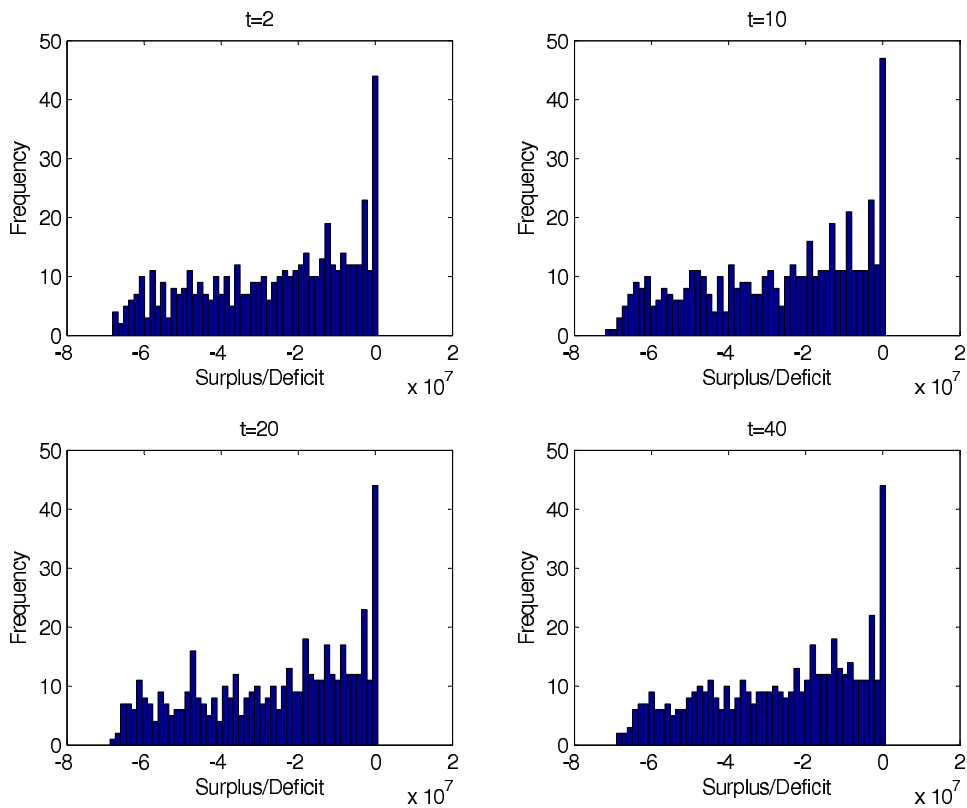


Figure 8: **LP Model Stress Testing High Interest Rate Surplus/Deficit Distribution**

Again we wish to look at the Solvency ratio and at the Sortino ratio of the SP, CCP and ICCP model during high and low interest rate scenarios. This is plotted in Figure 10 and in Figure 11. As expected the ICCP model has a Solvency ratio of zero in both cases, which suggests a perfect matching of assets and liabilities at all time periods. The SP model has a desired solvency ratio of zero only in the low interest rate scenarios, while during high interest rates, the assets outperform the liabilities. The CCP model has solvency ratios close to zero during high interest rate scenarios and the assets outperform the liabilities during low interest rates. Looking now at the Sortino ratio the CCP model has an upward sloping curve for both high and low interest rate scenarios. The SP behaves similar

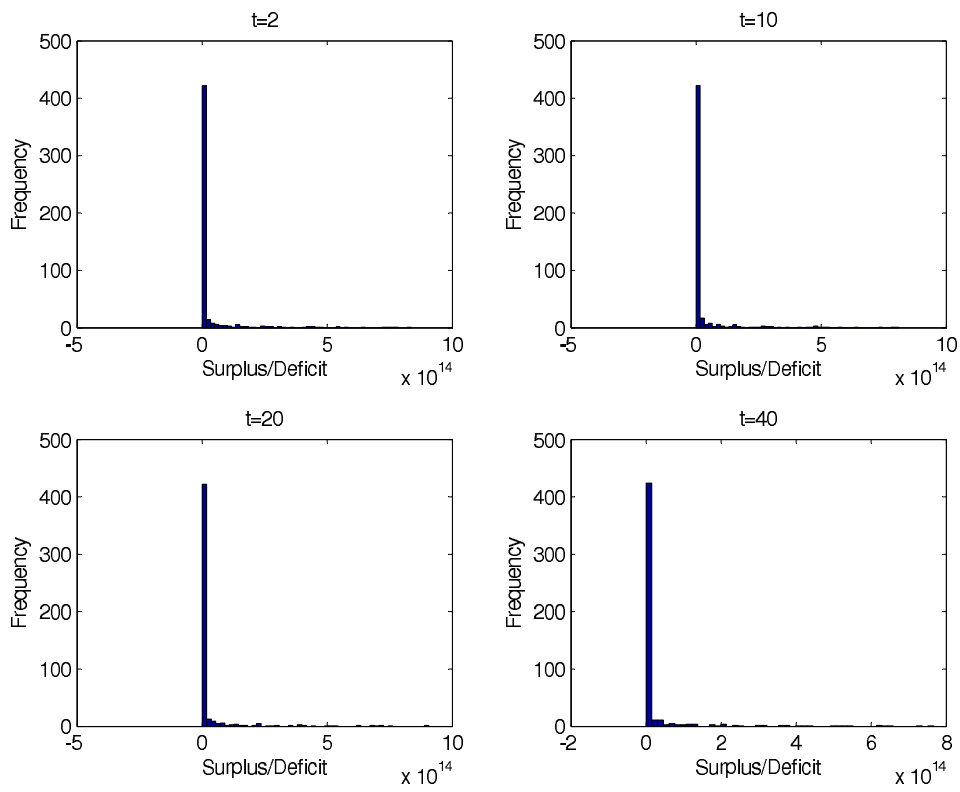


Figure 9: LP Model Stress Testing Low Interest Rate Surplus/Deficit Distribution

during high interest rates, while it is close to zero during low interest rates, which means again a close PV match of the assets and liabilities. The ICCP model is close to zero during low interest rates but not as close as the SP and during high interest rates it gets closer to zero the longer the time period, while the other models get further away from zero for longer time periods.

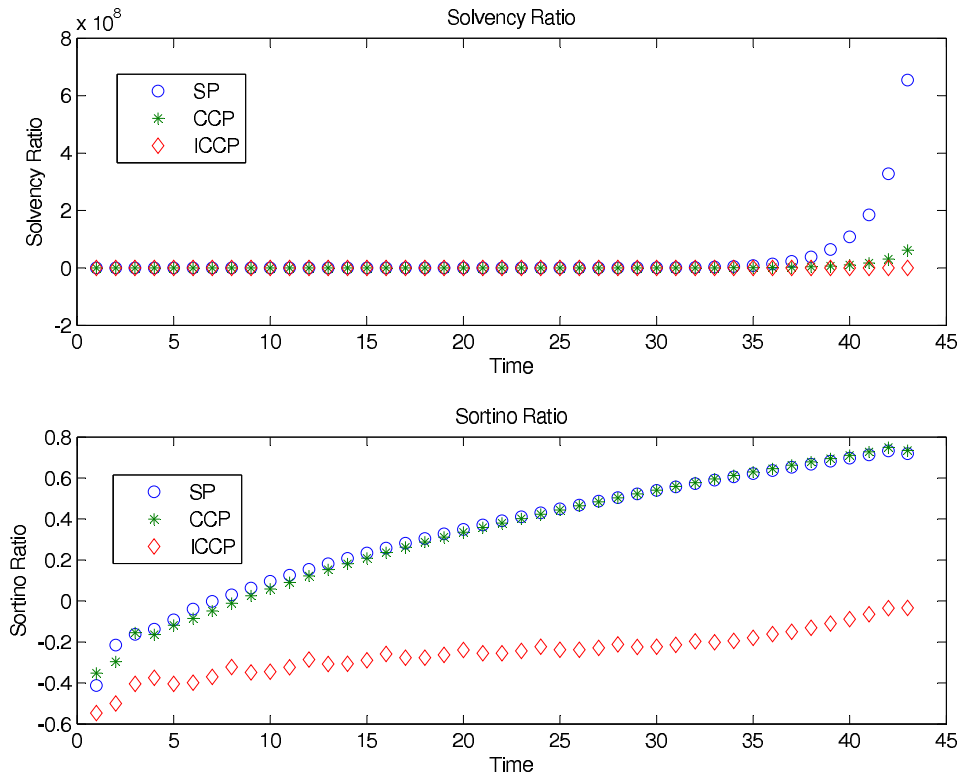


Figure 10: **Solvency and Sortino Ratios for Stress Testing (High Interest Rates)**

Table 2 shows the t-test statistics for the three null hypothesis for each ratio during high interest rates. For the ICCP Solvency and SP, CCP and ICCP Sortino ratio there is no evidence at all to accept the null hypothesis, while at 6% (2-tailed) we accept the null hypothesis for the SP Solvency and CCP Solvency ratio to have a mean of zero.

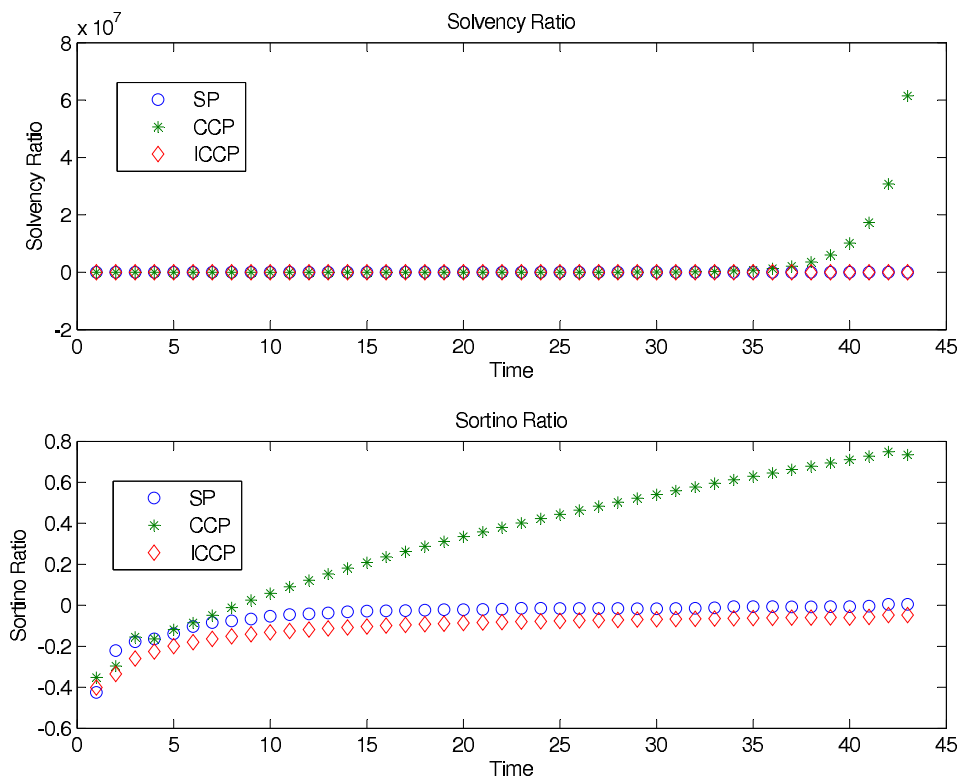


Figure 11: Solvency and Sortino Ratios for Stress Testing (Low Interest Rates)

Table 3 shows the t-test statistics for the three null hypothesis for each ratio during low interest rates. For the SP Solvency and SP, CCP and ICCP Sortino ratio there is no evidence at all to accept the null hypothesis, while at 6% and 81.8% (2-tailed) we accept the null hypothesis for the CCP Solvency and ICCP Solvency ratio to have a mean of zero.

T-Test

One-Sample Statistics

	N	Mean	Std. Deviation	Std. Error Mean
SPSolvency	43	33354485	113376037.92	17289684
SPSortino	43	.33879393	.294125666	.04485374
CCPSolvency	43	3134089.2	10653148.776	1624590.0
CCPSortino	43	.32874558	.307187566	.04684566
ICCPSolveny	43	-.75521393	.392816998	.05990403
ICCPSortino	43	-.26235725	.106371994	.01622158

One-Sample Test

	Test Value = 0					
	t	df	Sig. (2-tailed)	Mean Difference	95% Confidence Interval of the Difference	
					Lower	Upper
SPSolvency	1.929	42	.060	33354484.6	-1537510	68246479
SPSortino	7.553	42	.000	.338793930	.24827541	.42931245
CCPSolvency	1.929	42	.060	3134089.18	-144466.1	6412644.5
CCPSortino	7.018	42	.000	.328745581	.23420720	.42328396
ICCPSolveny	-12.607	42	.000	-.755213935	-.87610516	-.63432271
ICCPSortino	-16.173	42	.000	-.262357254	-.29509372	-.22962079

T-Test

One-Sample Statistics

	N	Mean	Std. Deviation	Std. Error Mean
SPSolvency	43	17.645652	5.844688319	.89130661
SPSortino	43	-.04804151	.077540377	.01182480
CCPSolvency	43	3134089.2	10653148.776	1624590.0
CCPSortino	43	.32874558	.307187566	.04684566
ICCPsolvency	43	-.00208792	.058997393	.00899702
ICCPSortino	43	-.11053723	.075113048	.01145463

One-Sample Test

	Test Value = 0					
	t	df	Sig. (2-tailed)	Mean Difference	95% Confidence Interval of the Difference	
					Lower	Upper
SPSolvency	19.798	42	.000	17.6456518	15.846922	19.444381
SPSortino	-4.063	42	.000	-.048041507	-.07190491	-.02417810
CCPSolvency	1.929	42	.060	3134089.18	-144466.1	6412644.5
CCPSortino	7.018	42	.000	.328745581	.23420720	.42328396
ICCPsolvency	-.232	42	.818	-.002087916	-.02024463	.01606880
ICCPSortino	-9.650	42	.000	-.110537229	-.13365361	-.08742085

5 Conclusions

This report showed how to measure and evaluate portfolio performances. A collection of risk-adjusted measurements from both academia and industry were explained; of which two of them were used to evaluate a stochastic asset and liability management model: namely the Sortino ratio and the Solvency ratio. By using hypothesis testing, which is a classical statistical method the outcome of the ratios is further examined.

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